

Variations and Clearance Computation in Overconstrained Mechanisms

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Abstract

Due to manufacturing deviations, introducing clearance is needed to meet both assembly and mobility requirements of the mechanism and make it actually work. However, large clearance influences the mechanism's accuracy. A wise clearance computation is then crucial. In the present paper, we propose to explicitly determine the minimum clearance values necessary for both assembly and mobility of the system. Compatibility relations will first be determined based on a vectorial modelling of a mechanism and will be used in tolerances and clearance computation. Two concepts will also be defined to help minimising clearance values, ideal mechanism concept and associated mechanism concept. These concepts will be illustrated on a 3D case study: the Bennett linkage.

Keywords: *Overconstraint, Clearance computation, Minimum clearance, Tolerancing, Mechanism, Assembly requirement, Mobility requirement.*

1. Introduction

During the design process, tolerance allocation deserves a great attention since it determines the involved manufacturing processes and tools, as well as the clearance values. Hence, it highly impacts both the cost and the performance of the mechanism. Overconstrained mechanisms deserve a greater attention since they need particular geometric conditions for their good functioning.

However, tolerance analysis and synthesis are usually performed in a particular position of the mechanism. In this paper, it will be shown that, for an example of overconstrained mechanism, deviations values computed in a position may be inadequate for other positions of the mechanism's mobility cycle, in which case they only guarantee the assembling of the mechanism in that position and neither its mobility nor its assembling in other

positions. Due to manufacturing deviations, there is a need to introduce clearance to make the mechanism actually work. However, large clearance influences the mechanism's accuracy. In the present paper, we propose to explicitly determine the minimum clearance values necessary for both assembly and mobility of the system. For this purpose, first a vectorial modelling, based on the TTRS (Topologically and Technologically Related Surfaces) concept, is presented which enables to generate a set of relations between the dimensional parameters of each part and the movement parameters of each joint. The generated equation system describes the mechanism. Analysing this equation system leads to a set of compatibility equations showing the dependence between the variations on the specification parameters helping tolerancing synthesis. Clearance values can then be deduced. Two concepts will also be defined, firstly the ideal mechanism concept and secondly the associated mechanism concept. These concepts will be illustrated on a 3D case study: the Bennett linkage. It will be shown that these concepts help minimizing the clearance values.

2. Compatibility relations between specification parameters

2.1 Geometric specifications

During the design stage, an engineer wishes to characterize a mechanism by a set of functional parameters called "specification parameters". These parameters are distances or angles between geometric elements, mechanical resistance, speed, acceleration, mass or cost, etc... Unfortunately, for several reasons, these specification parameters chosen by the designer do not define univocally the required mechanism. There are sometimes too few specifications, in this case the mechanism is not fully defined, or too many specifications, in this case the mechanism is functional only when parameter values are inter-compatible. In other cases, specification parameters appear to be

independent but are not, and the mechanism cannot, therefore, be constructed. In the following pages, we are particularly interested in geometrical parameters of overconstrained mechanisms in order to determine the compatibility relations and then compute the clearance to be introduced in the joints.

2.2 Geometric modelling

The geometric modelling used here is based on the TTRS concept (Topologically and Technologically Related Surfaces) [1], [2]. Using the mathematical structure of the displacement set, Clément Rivière and Temmermann [1] have proven that all the surfaces can be exhaustively classified into seven elementary surface classes: a spherical surface, a planar surface, a cylindrical surface, a helical surface, a rotational surface, a prismatic surface and "any" surface. To each class of surfaces is associated, at least, a subset of the set of displacements that keeps the surface invariant. To each elementary class of surfaces, geometric elements (point, line, and plane) can be associated, which are called MGDE (Minimum Geometric Datum Element). When two or more elementary surfaces are combined, the resulting TTRS can be classified into one of the seven classes mentioned above. From a combinatory point of view, there are 28 possible associations of surfaces. If we consider each case of relative position between the combined surfaces, we obtain 44 reclassification possibilities. The relative position is described by constraints on the MGDE. Clément *et al.* [2] define 13 possible constraints between geometric elements (point, line and plane). These constraints are perpendicularity, parallelism, angle, distance and coincidence constraints. To model a mechanism, functional surfaces are first identified. The MGDE relative to each surface is determined and the mating conditions are translated into constraints from the 13 constraints between geometric elements. We consider the set of specification (geometrical) parameters composed of the specified angles and distances between that

will be noted S . A vectorial modelling is then performed to model the geometric elements and constraints using a set of modelling parameters Q which, by definition, forms a complete, consistent, minimal system. By writing the equivalence of the two sets of parameters, s_i and q_i , we will deduce the completeness and consistency of the specification as well as the clearance required. This vectorial modelling based on the TTRS has already been presented in [2] and [3]. Other modellings are possible; note, for example, those developed in [4], [5] and [6]. A set of m equations can be established that depicts the relation between specification parameters and modelling parameters (1). These equations are divided into two sub-systems: the first one establishes the relation between the specification parameters and the modelling parameters and the second one characterizes the loop closure equation.

$$\begin{cases} T(Q) - K(S) = 0 \\ B(Q) = 0 \end{cases} \quad (1)$$

Where T and B are functions of the modelling parameters $Q = (q_1, q_2, \dots, q_p)$ and K is a function of the specification parameters $S = (s_1, s_2, \dots, s_n)$.

2.3 Compatibility relations for assemblability requirement

In this study, we are only interested in overconstrained systems and propose a method to establish the compatibility relations between specification parameters. It's well known that, in an overconstrained mechanism, all the dimensions are not independent. Moreover, due to manufacturing errors, deviations from the nominal values are noted on the real dimensions. So, these deviations shall also be dependant from one another and must satisfy some relations, named as "compatibility relations", in order to ensure the correct assembling and functioning of the mechanism. These relations are often difficult to express in general terms; on the other hand, they

are simple to determine for a specific position. Many researchers attempted to obtain these relations. For this aim, different approaches have been used. We name for instance the use of static equations [7], small displacement screws [4] or inverse kinematics equations [8]. In this work we suggest to determine the compatibility relations around a specific position by differentiation of (1) and discussion of the linear system thus obtained. See [9], [10] and [11].

The equations system (1) is equivalent to a system of the form:

$$F(Q, S) = 0 \quad (2)$$

The equations system (2) after differentiation is written:

$$\frac{\partial F(Q, S)}{\partial S} \cdot dS + \frac{\partial F(Q, S)}{\partial Q} \cdot dQ = 0 \quad (3)$$

Where dS components represent the deviations on the specification parameters and dQ components represent the deviations on the modelling parameters.

A mathematical treatment of equations (3) leads to the compatibility relations for assemblability noted CA between the deviations on the specification parameters as follows (For further details please refer to [10]):

$$M_{CA} \cdot dS = 0 \quad (4)$$

The relations (4) show the dependence of the deviations on the specification parameters. When they are respected, they guarantee the studied mechanism to be assembled around the initial position but don't give any information about its mobility. The compatibility relations for mobility requirement will be dealt with in the next section.

2.4 Compatibility relations for mobility requirement

In this section will be presented a method to establish compatibility relations assuring both

assembling and mobility requirements. For this aim, we need to write more relations. According to [12] and [8], satisfying the assemblability relations in several positions is enough to assure the mobility of the mechanism. The number of the studied positions depends on the degree of mobility of the mechanism. The compatibility relations for mobility requirement are given by (5):

$$\mathbf{M}_{CM} \cdot dS = 0 \quad (5)$$

Where \mathbf{M}_{CM} is an association of k assemblability matrices \mathbf{M}_{CAi} , k is the number of the positions to be studied (6)

$$\mathbf{M}_{CM} = \begin{pmatrix} \mathbf{M}_{CA1} \\ \vdots \\ \mathbf{M}_{CAk} \end{pmatrix} \quad (6)$$

3 Clearance computation

Adding clearance in a mechanism's joints is useful to allow the interchangeability of the manufactured parts. However, large clearance affects the mechanism's accuracy. So to assist the designer in his tolerancing task, it is essential to know the biggest value of acceptable clearance which assures the correct functionality of the mechanism. Indeed, this limit quantity imposes the maximal dimensional variations which are acceptable for the manufacturing parts. The control of these values is essential during the products industrialization phase because the manufacturing cost is strongly linked with the wanted accuracy [13]. The aim of this paragraph is to define a "framework" of parametric tolerancing simulation for mechanisms and for assemblies. The objective of this tool is to assist designers during the determination phase of the acceptable variations of the manufacturing parts' dimensions. This framework is built on the results of two works. First, the vectorial modelling of the parts, assemblies and mechanisms; second the determination of the compatibility relations

between the variations of the specification parameters (results presented in the previous part). In fact, compatibility relations show the dependence between the deviations on the specification parameters. So knowing these relations, we can express a part of the deviations relatively to the others. For this purpose a transfer function is firstly defined. This function depends on the choice of the deviations to compute. Then, once the deviations are known, the corresponding clearance values can be determined.

3.1 Transfer function

The transfer function is determined relatively to a particular partition of the set of the specification parameters into input and output specification parameters sub-sets named respectively S_{in} and S_{out} . These sub-sets are such as their union contains the whole specification parameters while their intersection is empty $S = S_{in} \cup S_{out}$ and $S_{in} \cap S_{out} = \emptyset$. The corresponding deviations sub-sets are respectively dS_{in} and dS_{out} . The first sub-set contains the given deviations (they can be either measured or imposed by a technical or conceptual constraint). The second sub-set contains the h deviations to calculate relatively to the deviations given in the first sub-set.

In this case we can write that:

$$dS_{out} = -M_{C_{out}}^{-1} \cdot M_{C_{in}} \cdot dS_{in} \quad (7)$$

So we can define a transfer function FT that allows determining the unknown deviations relatively to the measured (or imposed) ones. The corresponding matrix is given by:

$$M_{FT} = -M_{C_{out}}^{-1} \cdot M_{C_{in}} \quad (8)$$

Let's remind the reader here that the treatment performed until now is only valuable around a given configuration (a position of the mechanism's mobility cycle). Thus, the transfer function itself is

relative to the studied configuration. For a given configuration, the transfer function is not unique and also depends on the chosen partition of the specification parameters set. The analysis of the compatibility relations in several configurations shows the evolution of the deviations during the mobility cycle of the mechanism. Taking into account the ranges of variation of the deviations throughout the whole mobility cycle when computing the clearance values allows to have an adequate mechanism that can be assembled and can function properly.

3.2 Clearance computation

The transfer function allows determining the unknown deviations relatively to the known ones (provided that there are as many unknown deviations as equations). If the deviations' ranges of variation are known, we'll be able to deduce the ranges of variation of the unknown deviations. In fact, the known deviations dS_{in}^s are usually given within ranges rather than precise values. Thus, thanks to the compatibility relations, we can deduce the ranges of variation of the unknown deviations.

3.3 Use of metric tensors

In the following we propose a method for clearance computation based on the use of metric tensors. The clearance could be computed even without the compatibility relations. During the functioning, the relative positions of the parts vary and can be hardly predicted. So, even in this method, we will proceed in a local way. We'll begin by building the actual parts independently by adding the deviations values. We'll then choose a layout for the parts in the assembling in order to determine the clearance in the joints locally too, considering each joint separately. The clearance values will be computed relatively to a target mechanism that can be either the nominal or the associated mechanism. The associated mechanism

is a new concept defined in this work in order to minimize the clearance values. It is inspired from the association methods employed in metrology field. In this domain, to identify the characteristics of a real shape, it is necessary to proceed in two stages: firstly, to know the shape class (plane, cylinder, sphere, etc.) of the measured element. This element is named "ideal element". Secondly: to find geometric parameters of an ideal element which is closer to the shape built with the measured points on the actual surface according to a chosen algorithm. This object is called "associated element". In the proposed approach, an "ideal mechanism" is so defined: it is a mechanism composed of "ideal" or "associated" parts and "ideal" joints (it means that parts are in contact). It possesses the same properties as the nominal mechanism (assemblability or mobility and degree of freedom).

3.4 Nominal and associated mechanism

An associated mechanism is defined as an ideal (without joint clearances) mechanism with the same degree of freedom than the nominal mechanism to which it is associated. Its dimensions slightly differ from the nominal mechanism's ones. Consequently, the variations of the specification parameter of an associated mechanism have to respect the compatibility relations CM .

Before continuing, it is necessary to clarify some notations: the i^{th} specification parameter of the j^{th} part of the nominal mechanism is called sp_{ij}^{nom} ; the i^{th} specification parameter of the j^{th} part of the associated mechanism is called sp_{ij}^{ass} . These two types of parameters respect the relations (9):

$$sp_{ij}^{ass} = sp_{ij}^{nom} + dsp_{ij}^{nom} \quad (9)$$

With dsp_{ij}^{nom} respecting the compatibility relations CM . It is illustrated on the Figure 1.

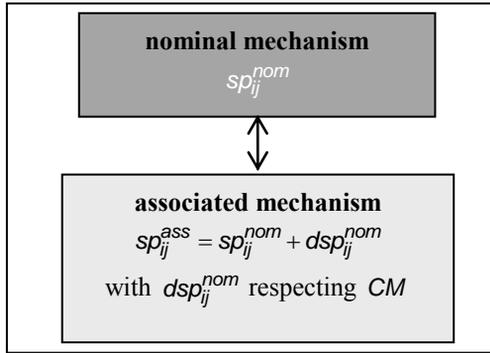


Figure 1 : Nominal and associated mechanism

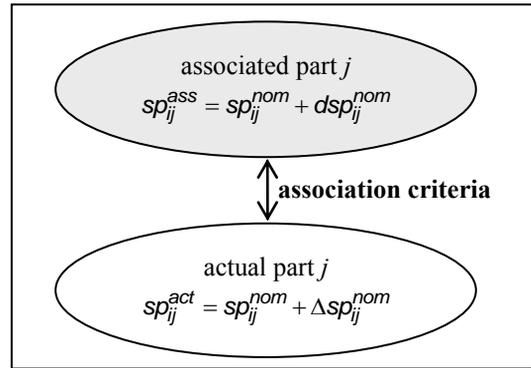


Figure 2 : Actual and associated parts

3.5 Actual and associated parts

The actual part is a model of the manufactured part. The form defects are not taken into account here. The dimensions of this part are measured and the deviations with regard to the nominal dimensions are determined. This deviation between a measured dimension and the corresponding nominal dimension is named Δ . Also, the deviation of the i^{th} specification parameter of the j^{th} actual part is called $\Delta sp_{ij}^{\text{nom}}$. In the same way, the i^{th} specification parameter of the j^{th} actual part is called sp_{ij}^{act} .

These two parameter types respect the relations (10):

$$sp_{ij}^{\text{act}} = sp_{ij}^{\text{nom}} + \Delta sp_{ij}^{\text{nom}} \quad (10)$$

Note that deviations depend on the manufacturing process of parts. They can take different values. The possible dependency relations between these deviations results from the behaviour of the used machine tool and are not considered here. The relative position of the actual parts with regard to the associated parts is calculated by one of the techniques of association used in the metrology field. We can give as an example, the methods using the small displacement torsor [14] or the variations of the specification parameters [15]. In the proposed method, it is possible to choose various association criteria. It is illustrated on the Figure 2.

3.6 Clearance computation

The first step in the clearance computation is to determine the target mechanism. It can be either the nominal mechanism or an associated mechanism. The metric tensor of this mechanism is known as well as the lengths' vector. The metric tensor G_{Ci} of each target part is also known.

The second step is to build the metric tensor G_{Ri} of each actual part of the actual mechanism knowing the values of the deviations. Then, for each part, a metric tensor G_{CRi} is constructed which gives the angular association by defining the angular relations between the target (associated or nominal) mechanism's vectors and the actual mechanism's ones. The metric tensor defining the angular relations between both target and actual part's vectors is given by (11)

$$G_{C\&Ri} = \begin{pmatrix} [G_{Ci}] & [G_{CRi}] \\ [G_{CRi}] & [G_{Ri}] \end{pmatrix} \quad (11)$$

A Singular Value Decomposition of G_{Ri} gives $G_{Ri} = U_{Ri} \cdot S_{Ri} \cdot V_{Ri}^T$.

The tensor G_{Ri} being positive and defined, we have: $U_{Ri} = V_{Ri}$ and S_{Ri} is a diagonal matrix

containing positive or nul values. The metric tensor of the actual part can be written as (12):

$$G_{Ri} = (U_{Ri} \cdot \sqrt{S_{Ri}}) \cdot I \cdot (U_{Ri} \cdot \sqrt{S_{Ri}})^T \quad (12)$$

In the same way, the metric tensor of target part is given by (13):

$$G_{Ci} = (U_{Ci} \cdot \sqrt{S_{Ci}}) \cdot I \cdot (U_{Ci} \cdot \sqrt{S_{Ci}})^T \quad (13)$$

Thus, the metric tensor giving the angular relations between target and actual vectors is given by (14):

$$G_{CRi} = (U_{Ci} \cdot \sqrt{S_{Ci}}) \cdot I \cdot (U_{Ri} \cdot \sqrt{S_{Ri}})^T \quad (14)$$

Once the angular relations are defined, the next step consists in the affine association, meaning the choice of the position of the actual parts relatively to the target ones. Different choices are possible, for instance the actual and target bars may be coincident in a point M that could be the end, the middle or any point of the bar. In the following we choose to make the point M as the middle of the two bars

Figure 3. P_{ci2} is the intersection point of both neighbour parts in the target mechanism. P_{ri2} and P_{rj1} are end points of actual neighbour parts i and j respectively. So, for clearance computation between the surfaces of the joint, first the relative position of the MGDEs of respectively the actual and associated parts is determined. Then the clearance between the two surfaces of the actual parts of each joint is deduced.

The clearance between two neighbour actual parts is given by a clearance vector \vec{J}_{ij} given by (15):

$$\vec{J}_{ij} = \vec{M_i P_{ci2}} + \vec{P_{ci2} M_j} + \vec{M_j P_{rj1}} + \vec{P_{ri2} M_i} \quad (15)$$

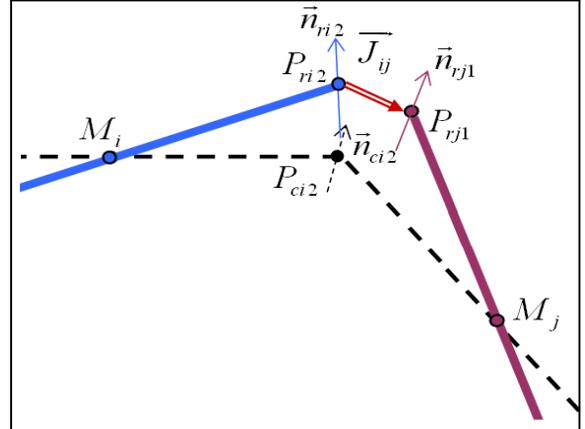


Figure 3: Positioning the actual parts according to the target ones

4 Case study: the Bennett linkage

4.1 Introduction

The proposed method will be illustrated with the Bennett linkage which is a three-dimensional 4R (having 4 revolute joints) overconstrained mechanism. It consists of four parts connected by means of four revolute joints whose axes are neither parallel nor concurrent (Figure 4). Each part is modelled by means of three unit vectors: two unit vectors carried by the revolute joints' axes and a unit vector carried by the common perpendicular. Each part is specified by a set of four parameters (a length and three angles). The command (input) angle θ is the angle of the revolute joint of axis \vec{n}_i (Figure 4).

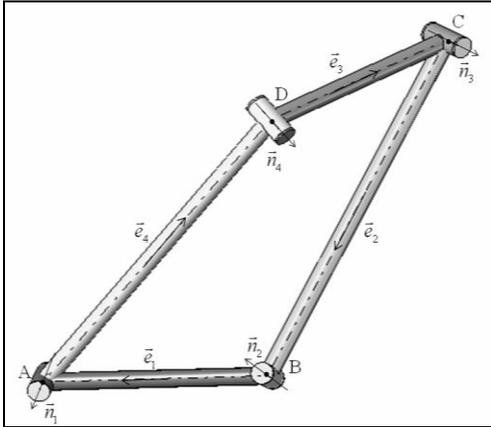


Figure 4 : The Bennett Linkage

The nominal dimensions of the studied Bennett linkage are given by Table 1:

Table 1: Nominal values of the specification parameters of the Bennett linkage

Bar 1	$L_1 = 100 \text{ mm}$	$\alpha_1 = 30^\circ$
Bar 2	$L_2 = \sqrt{3} \cdot 10^2 \text{ mm}$	$\alpha_2 = 60^\circ$
Bar 3	$L_3 = 100 \text{ mm}$	$\alpha_3 = 30^\circ$
Bar 4	$L_4 = \sqrt{3} \cdot 10^2 \text{ mm}$	$\alpha_4 = 60^\circ$

4.2 Compatibility relations

As explained above, there are two types of compatibility relations: compatibility relations for assemblability requirement (CA) and compatibility relations for mobility requirement (CM). The first ones are given for a unique position of the studied mechanism (for a command angle value) and differ from one position to another. For example, for a command angle value of ten degrees, the three compatibility relations of the studied Bennett linkage are given by (16):

$$M_{CA(\theta=10^\circ)} \cdot dS = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (16)$$

With:

$$dS = (dL_1 \quad dL_2 \quad dL_3 \quad dL_4 \quad d\alpha_1 \quad d\alpha_2 \quad d\alpha_3 \quad d\alpha_4 \quad d\theta)^T$$

$$M_{CA(\theta=10^\circ)} = \begin{pmatrix} -0.0017635 & -0.0085773 & -0.0027230 \\ -0.0015304 & -0.0085115 & -0.0030498 \\ 0.0014842 & 0.0084870 & 0.0031468 \\ 0.0016917 & 0.0085636 & 0.0028051 \\ -0.2052424 & 0.0934773 & -0.1734818 \\ 0.2177426 & -0.1073968 & 0.1424431 \\ 0.2536188 & -0.0778475 & 0.1000753 \\ -0.2338680 & 0.1021869 & -0.1179743 \\ 0 & 0 & 0 \end{pmatrix}^T$$

This set of relations is named $CA_{(\theta=10^\circ)}$.

When these relations are respected, the mechanism may be assembled in the neighbourhood of the initial position $\theta = 10^\circ$. Note that, in $M_{CA(\theta=10^\circ)}$ matrix, the coefficients of $d\theta$ parameter vanish. This proves that the dimensional variations of the bars cannot be "corrected" by the variation of the command angle. It is the general case of mechanisms with degree of freedom.

The CM relations are obtained by satisfying the CA for several positions. For the Bennett linkage, the number of studied positions is two. If we choose the command angle values of $\theta = 10^\circ$ and $\theta = 40^\circ$, the CM are given by (17):

$$M_{CM} \cdot dS = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (17)$$

$$\text{With: } M_{CM} = \begin{pmatrix} M_{CA(\theta=10^\circ)} \\ M_{CA(\theta=40^\circ)} \end{pmatrix}$$

And the numeric application gives:

$$M_{CA_{(\theta=40^\circ)}} = \begin{pmatrix} 0.001697799 & -0.001345219 & -0.004185093 \\ 0.000260857 & -0.001578249 & -0.004429132 \\ 0.000119427 & 0.001303927 & 0.004543269 \\ -0.001310033 & 0.001602089 & 0.004222339 \\ -0.138248395 & 0.249181681 & -0.154935314 \\ 0.06387168 & -0.30333083 & -0.034290065 \\ -0.176504394 & -0.242029733 & 0.092897323 \\ 0.041045916 & 0.300946848 & 0.054969395 \\ 0 & 0 & 0 \end{pmatrix}^T$$

The rank of M_{CM} matrix is five. It is thus possible to calculate five parameters, suitably chosen, knowing the four remaining ones.

4.3 Clearance computation

In the following, we calculate the clearance values in an actual Bennett linkage having deviations according to the nominal values. For this, arbitrary deviation values are considered that are give in Table 2.

In the first step, the target mechanism on which the actual parts will be positioned is the nominal one. The affine association is such as the actual and target bars are coincident in their midpoint.

Table 2 : Deviations of the actual parts

Bar 1	$\Delta L_1 = 0.11 \text{ mm}$	$\Delta \alpha_1 = -0.8^\circ$
Bar 2	$\Delta L_2 = -0.15 \text{ mm}$	$\Delta \alpha_2 = -0.9^\circ$
Bar 3	$\Delta L_3 = 0.2 \text{ mm}$	$\Delta \alpha_3 = 1.1^\circ$
Bar 4	$\Delta L_4 = -0.1 \text{ mm}$	$\Delta \alpha_4 = -1^\circ$

The amplitude of the clearance vector \vec{J}_{ij}

Figure 3. for input angle values from 10° to 90° are given by Figure 5.

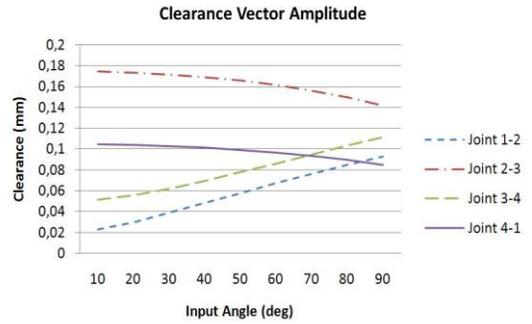


Figure 5 : Clearance vector amplitude using the nominal mechanism as reference

We note that for different input angle values (from 10° to 90°), the clearance vectors' amplitude varies. For input angle of 10° , the clearance vector amplitudes for joints 1_2 and 3_4 are around 0.05 mm and 0.02 mm respectively while for joints 2_3 and 4_1 the clearance amplitudes are smaller and are around 0.18 mm and 0.1 mm respectively. As the input angle increases, the clearance in joints 1_2 and 3_4 increases to reach around 0.11 mm and 0.09 mm respectively, while it decreases in joints 2_3 and 4_1 to reach around 0.14 mm and 0.08 mm respectively. If clearance is computed using a target mechanism different from the nominal one, what we called an associated mechanism, the clearance values are not the same and are likely to get minimized if the target mechanism is well chosen. For the same actual parts with deviation values in Tabel 2, we will consider an associated mechanism having the variations given in Table 3.

Table 3: Variations of the associated parts

Bar 1	$dL_1 = 0.1 \text{ mm}$	$d\alpha_1 = -0.35^\circ$
Bar 2	$dL_2 = -0.15 \text{ mm}$	$d\alpha_2 = 1.2351828^\circ$
Bar 3	$dL_3 = 0.1 \text{ mm}$	$d\alpha_3 = -0.35^\circ$
Bar 4	$dL_4 = -0.15 \text{ mm}$	$d\alpha_4 = 1.2351828^\circ$

Let's keep on mind that the associated mechanism is an ideal mechanism. Thus the variations of the associated mechanism are chosen such as they satisfy the CM relations. Some deviations are chosen and the others are deduced using the CM compatibility relations. The amplitude of the clearance vector \vec{J}_{ij} for input angle values from 10° to 90° are given by Figure 6.

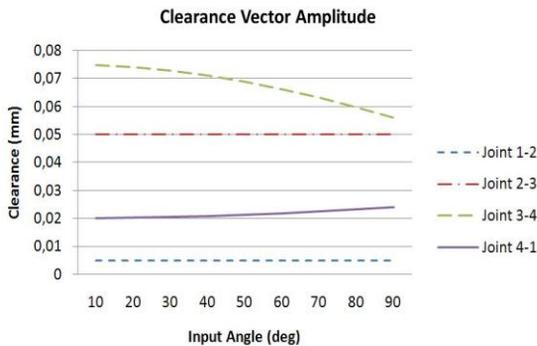


Figure 6 : Clearance vector amplitude using the associated mechanism as reference

We note that for this case, the maximal clearance values are about two times smaller with the use of an associated mechanism even though the variations of the associated mechanism were arbitrarily chosen. However, for some other test cases using an associated mechanism, the clearance values slightly increased or decreased.

5 Conclusions

In the first part of this paper, a method to give the compatibility relations for overconstrained mechanisms was presented. Two kinds of compatibility relations are presented: compatibility relations for assemblability requirement, available only around a position of the mechanism and compatibility relations for mobility requirement. In the second part, a method for clearance computation was presented. To help minimizing clearance, two concepts were defined: “ideal mechanism” concept and “associated mechanism” concept. It was shown that using an associated mechanism instead of the nominal one may lead to smaller clearance values and thus enhance the accuracy of the mechanism. In this paper, the clearance was characterized by the amplitude of the clearance vector between two neighbour parts. In the same way, the angular deviation between the joints’ axes of two neighbour actual parts can be easily determined to characterize the clearance. The choice of the associated mechanism as well as the association criteria can also be investigated to improve the clearance minimizing.

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