Defining acceptance criteria on parts in order to maximise the use of functional condition domain

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Abstract
Designers have at one's disposal several methods to calculate tolerances within dimension's chains and can specify several statistical indicators. The worst case method can conduct to over quality, quadratic often conducts to insufficient quality. Efficiency of other candidate methods are studied below and a new one is proposed, with the target to guarantee the functional condition at an acceptable level and allow the largest variation area on the components.
A simple solution consists in assigning to the parts the usual worst case tolerance range, and in verifying it regarding the $Cpk_{target} / \sqrt{(\text{lenght of dimension chain})}$

Keywords*: statistical tolerancing, tolerance analysis, simulations, inertial*

1 Introduction

Tolerance analysis and statistical dimensioning are implemented inside generalist tools or often custom tools. Designers use them as "black box", but in some cases have at one's disposal several methods to calculate tolerances within dimension's chains and can specify several statistical indicators for verification.

They become responsible for finding the compromise between "severity" and "risk", with impacts on cost and drawbacks on quality, in a context where the statistical hypothesis are clear only for experts.

- Worst case method gives the smallest tolerance interval on the components, in an industrial production it conducts often to over quality requirements.
- Quadratic method conducts to quality issues when some prerequisites are forgotten.
- Designers question about alternative methods: semi-quadratic, probabilistic, and a new one: inertial [1]

Designers are rarely specialists in statistics and expect guidance to choose the model appropriate to the situation.

The targets of this study are:

- To present some of the current tolerancing models[1] with their efficiency characteristics
- To propose a guideline to define the tolerance area suiting efficiency targets [2]
- To propose a practical method with target indicators.

2 Methods used for the study

2.1 *Reminder: basic rules*

Let us consider a dimension chain in an assembly on which a functional condition (FC) is defined. It is a characteristic expressed as a function of "n" elementary characteristics: $FC=f(X_1,X_2,..X_n)$

If the chain is linear, $FC=\Sigma(\alpha_i X_i)$ in which α_i are -1 or +1.

On the links (X_i), random variables, defining this chain, it is well known that:

- deviations from the nominal add together,
- variances add together,

as long they are independent.

Normality is not required.

[1] Probabilistic is kept out of scope (refer to appendix)

Vijay Srinivasan [3] proposed the following representation where δ is the deviation of each link from the nominal and σ^2 their variance.

$\delta=E(X_i)-VC$ in which $E(X_i)$ is the mean and VC the centred nominal value for the X_i characteristic. [5]

$\sigma^2=E\{[X_i-E(X_i)]^2\}$ [5]

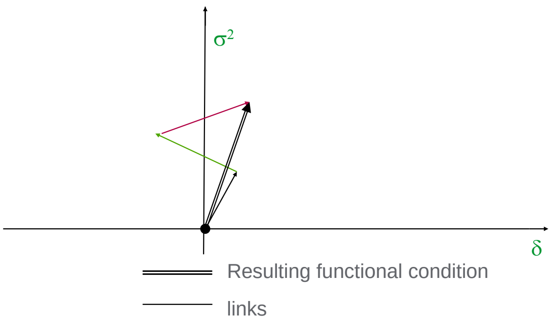

Figure 1: Srinivasan domain

As direct consequence, the deviation and the variance of the resulting functional condition are obtained by the sum of all link vectors.

2.2 *Statistical tolerance zone (STZ)*

In the following step we have to define the surface area corresponding to acceptance zone. Let's start from the functional requirement. The functional condition (FC) is usually submitted to one or 2 functional limits (USL, LSL); if we assume a direct correlation to the service functions, the acceptance criteria is then a maximum **non conforming rate** (NCR).

If we neglect the fact that we can have in the same time a both sided non conforming rate, and with a strong assumption regarding the normality of the distribution, the acceptance criteria is simplified and can be transformed in **Cpk**.

$Cpk=\min(USL-\mu;\mu-LSL)/3\sigma$

We give up the Srinivasan representation (δ, σ^2) and use a (δ, σ) coordinate system. [PA Adragna][4] On figure 2,

- TR is the Tolerance range defined by the functional limits=USL-LSL
- k is a ratio related to the Cpk target k= 3.Cpk target

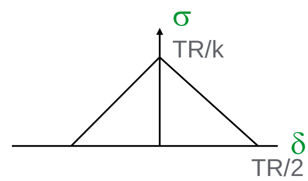

Figure 2: statistical tolerance zone (FC) In the following text, X axis will be called "deviation" axis and Y "variance" axis[2].

2.3 *Breaking down the STZ on the links (parameters)[3]*

The core of the problem is here: from STZ of the functional condition, define the appropriate STZ on the links. All batches (δ, σ) will combine according 2.1.

The French standard XP E04-008 [5] has published rules for:

- Arithmetical (worst case)
- Quadratic
- Semi-quadratic
- Inertial

Arithmetical, quadratic, semi-quadratic consist in defining a **Tolerance Range** on each elementary characteristic.

Inertial defines a **Tolerance on the Inertia.** As soon as a TR is defined, the STZ is adapted according to the standard, or for the moment, according to the habits.

2.4 *Simulation hypothesis*

We have done simulations for a 8 parameters dimension chain.

$n=8$

To simplify, links are identical but independent.

The FC Tolerance range (FC_{TR}) is +/- 0.5 mm. An hypothetical quality requirement 32 ppm on the FC is converted to a Cpk 1.33.

$Cpk_{FC}=1.33$

[2] We do not call it "standard deviation" axis as we should, in order to avoid confusion with X axis

[3] In the next chapters, X_i elementary characteristics will be called "parameters" or "links" according to the context (geometrical or mathematic)

- The mean deviation inside the STZ is assumed equiprobable.
- The standard deviation is the **maximum** allowed by the STZ.
- The resultant is calculated according 2.1
- 1000 run have been done for each parameter.
- Result is displayed, and we calculated the failure rate.

Notice that the simulations are pessimistic, because in an assembly is not usual to have all parameters at their limit (Cpk or else) together.

3 Application on different tolerancing methods

3.1 *Arithmetical*

Although statistical acceptation is not necessary (attribute or GO-NOGO is sufficient) it is more comfortable to produce statistics in order to avoid 100% inspection or an inspection based on the binomial law. In that case, a STZ derived from Cpk criteria is the most used. For a 8 links chain, TR parameter is calculated as follow:

$TRx=+/-0.0625=FC_{TR}/8$

Cpk target: 1.00 *(not justified)*

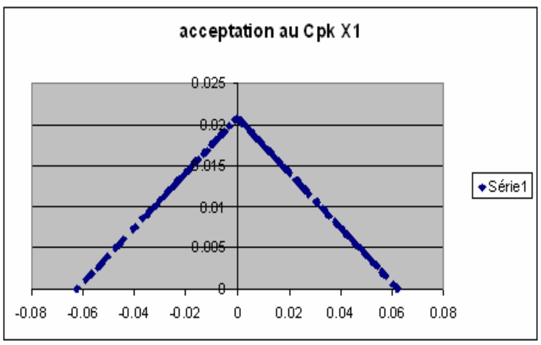

Figure 3: parameter STZ for arithmetical

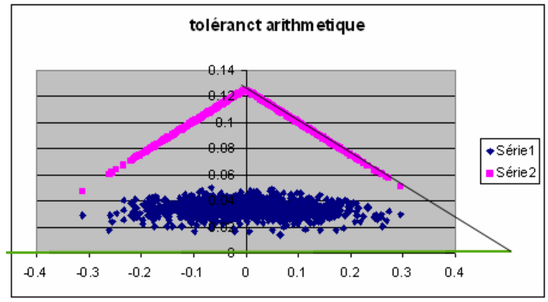

Figure 4: FC STZ arithmetical simulation result

Remarks: although parameters are sampled on the limit of their STZ (on the triangle) we get a point cloud; the reason is that the deviations δ can be compensated together. If we sample only on the same side on the parameter STZ, we get a curve inside the FC STZ, result of the Minkowski sum. [2]. In the other hand, if we sample inside the full parameter STZ, we will get points closer from the horizontal axis.

Findings:

- FC is satisfied,
- The STZ is not fully used, especially in variance.

This well known feature leads to quadratic calculation.

3.2 *Quadratic*

A Gaussian hypothesis is the usual unique assumption.

$TRx=FC_{TR}/\sqrt{(n)}/cpk_{FC}=+/-0.1326$

Usually, we use only the Cpk for acceptance criteria.

With a Cpk target=1.00 (consistent with the definition of TRx) we get:

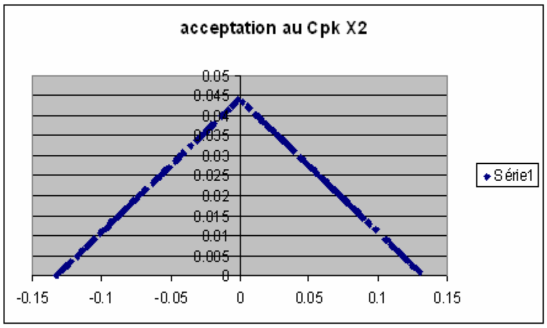

Figure 5: parameter STZ for quadratic

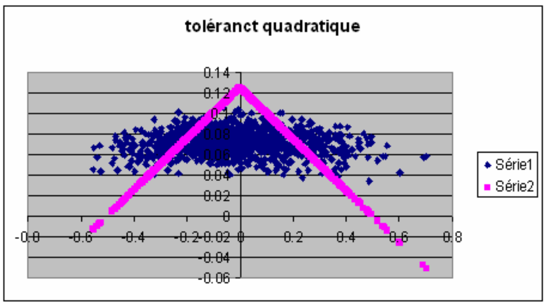

Figure 6: FC STZ quadratic simulation result

Findings:

- FC is not satisfied, especially in deviation.

The mean of FC is out of the range in 1.2% of cases (meaning more than 50% of NCR) and Cpk is not compliant in 35% cases. This well known feature leads to evaluate semi-quadratic or inertial calculation. NF X 04-008 recommends limiting the parameter deviation δ to 1 sigma (refer to appendix).

3.3 *Semi-quadratic*

The parameters have a range in which the mean can vary freely, and the remaining part of TR allows a Gaussian variation [6]. The tolerance on the FC consists in the arithmetical sum of the mean's ranges, and a statistical part calculated by quadratic way. Let us take a mean range of 60% inside TRx.

Remaining 40% correspond to 6 σ_x

Reverse calculation leads to TRx=+/-0.07926

The parameter STZ should be defined by a rectangle, (width equal to TRx*0.6) and constant height, because we assume in the calculation a constant Cp whatever the value of the mean. Let us look to a STZ defined by a limited mean deviation, and a Cpk:

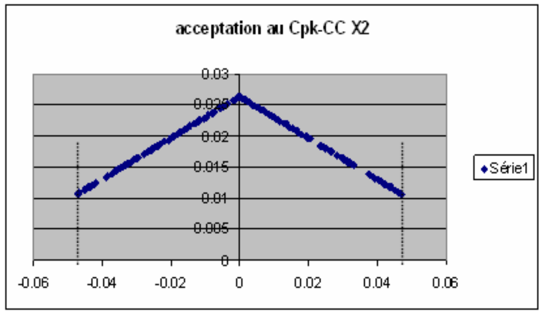

Figure 7 : parameter STZ for semi-quadratic

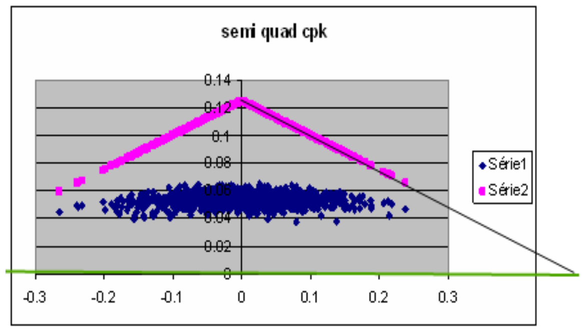

Figure 8 : FC STZ simulation for semi-quadratic

Findings:

- FC is satisfied,
- The STZ is not fully used, in deviation axis as in variance.

With the above 0.6 setting, we may observe that Cp requirement is not useful to ensure the quality.

3.4 *Inertial (regular)*

The calculation of the inertia tolerance on the parameters is consistent with the quadratic method: The change is related with the STZ, and consists in limiting the deviation at 1 sigma:

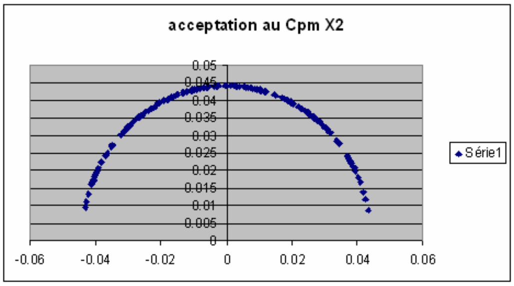

Figure 9 : parameter STZ for inertial

Remark: Cpm requirement on quadratic calculated TRx conducts to same STZ.

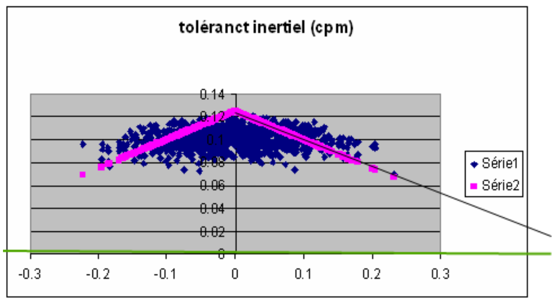

Figure 10: FC STZ simulation for Inertial

Findings:

- FC is not fully satisfied.

24% of Cpk are not compliant.

Remarks: non compliant points are not very far from the limit, and come from simulations done with all Cpk at the limit. The failures do not come from the deviation, that is mastered, but from the variance.

3.5 *Inertial, "corrected"*

NF E 04 008 gives a solution to satisfy fully the FC, according the Cpk $_{FC}$, through an adjustment related to *n* (length of the chain).
As result, the parameter STZ is slightly reduced:

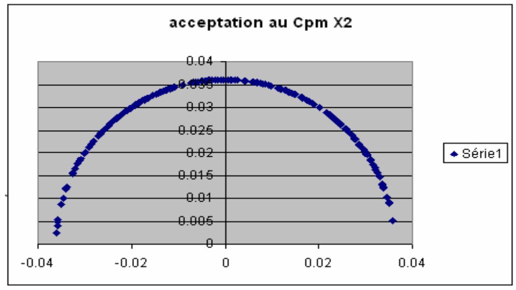

Figure 11: parameter STZ for inertial "corrected"

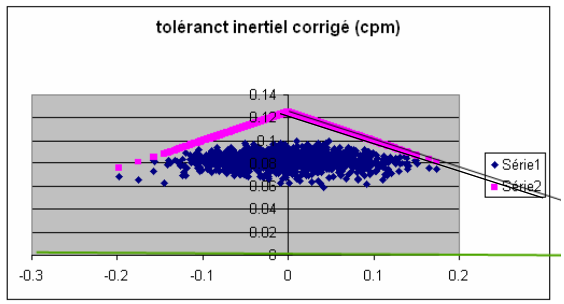

Figure 12 : FC STZ simulation for Inertial "corrected"

Findings:

- FC is successfully satisfied,
- The STZ is not fully used, in deviation axis.

3.6 *Simulations with n =4*

With a shorter dimension chain, findings are identical.

3.7 *Conclusion for the simulations*

According the above models:

- Either the Functional condition is not satisfied,
- Either the Statistical Tolerance Zone of the functional condition is not fully used.

Is it impossible to increase the efficiency, while controlling the risks on the FC ?

4 Synthesis of the STZ for parameters

Let us superpose the parameter's STZ of the previous methods:

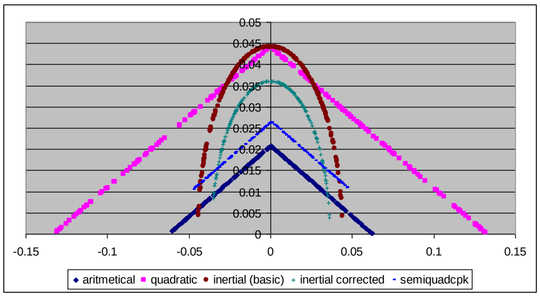

Figure 13: parameter STZ comparisons for 8 links

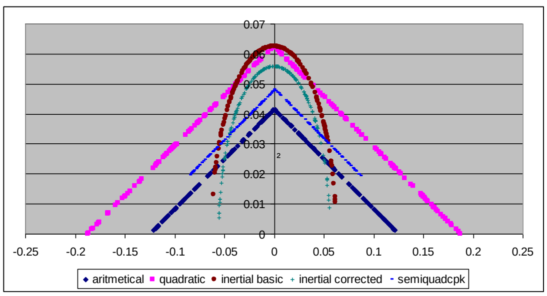

Figure 14 : parameter STZ comparisons for 4 links

Reminders from FC STZ simulations:

- Quadratic lets the FC deviate too much,
- Inertia corrected limits too much the deviation,
- Arithmetical limits too much the variance,
- Semi quadratic limits both.

5 Evidences:

- The maximum possible variance is defined by quadratic and inertia (regular) methods, as soon there is no deviation,
- The maximum deviation is given by arithmetical calculation…as soon there is no variance.

This defines two remarkable points in the STZ domain:

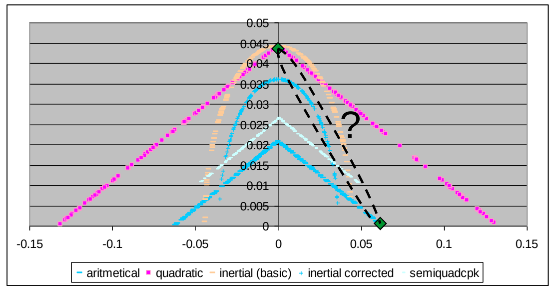

Figure 15 : parameter STZ remarkable points (8 links)

Questions:

What is the ideal curve going through these points?

- The right way is to break down the FC STZ through a "Minkowski division"[4].
- Simulation can produce approximations, with the target to satisfy the FC and to "fill" the FC STZ as much as possible.

6 Searching the ideal STZ

6.1 *Drafting Cpk STZ*

The first test consists in drafting a straight line, i.e. a Cpk domain:

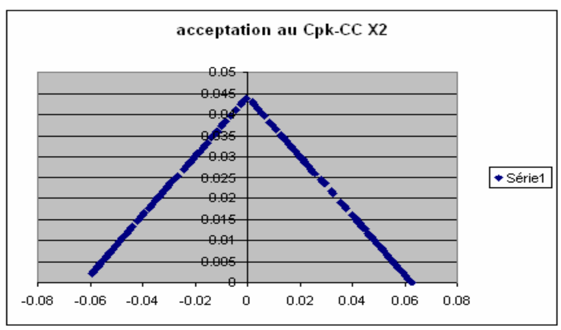

Figure 16: parameter STZ low cpk (8 links)

[4] Indeed the Functional condition STZ is obtained by "Minkowki sum" of all parameters STZ [2]. It could be possible to search which shape must have the parameter STZ to give the expected FC STZ for dimension "n"

Cpk=1/√(n) here n=8, so Cpk target=0.47

- Please notice that the Tolerance range is the arithmetical one.
- Constraint: the tolerance range in CAD is not sufficient to define the verification procedure, we need to associate the Cpk target value.

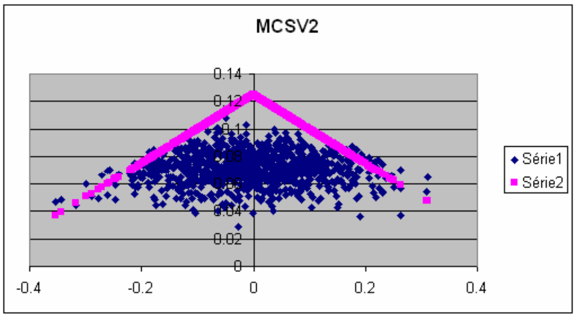

Figure 17: FC STZ simulation for Low Cpk (8 links)

Findings:

- FC is not fully satisfied,
- The STZ is not fully used, in deviation .

Cpk non conform rate:	3.7%
Average ppm rate:	7.1 ppm
Average ppm rate of non conform Cpk:	170ppm
Maximum ppm rate:	968 ppm

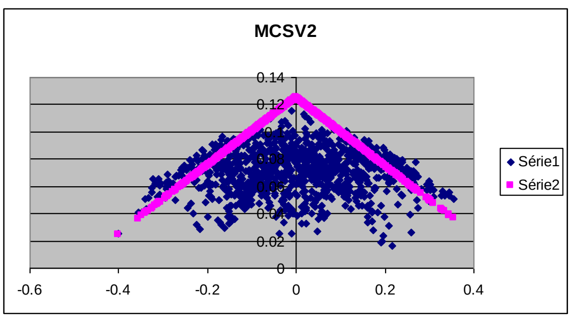

Figure 18: FC STZ simulation for Low Cpk (4 links)

Findings:

- FC is not fully satisfied,
- The STZ is almost fully used.

Cpk non conform rate:	18%
Average ppm rate:	69 ppm
Average ppm rate of non conform Cpk:	350 ppm
Maximum ppm rate:	6500 ppm

Remarks: not conform points are close to the limit, and come from simulations done with all Cpk at the limit.

6.2 *Optimizing the curve*

6.2.1 Aims

We have 2 aims:

- delete the external points to the domain; they come from highest variance when the deviation is important,
- expand the deviation possibility when the chain is long, to improve FC STZ efficiency.

The first aim can be favoured through a "variable Cpk", i.e. low Cpk when deviation is low, high Cpk when deviation is high.

The second one can be reached through a power function (n^p) attached to TRx calculation, in order to increase it when n is high (i.e. making an acceptable compromise between /n and /√(n)).

6.2.2 Improving the FC quality

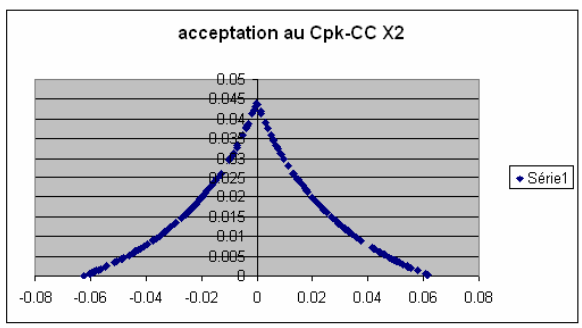

Figure 19: variable Cpk STZ (8 links)

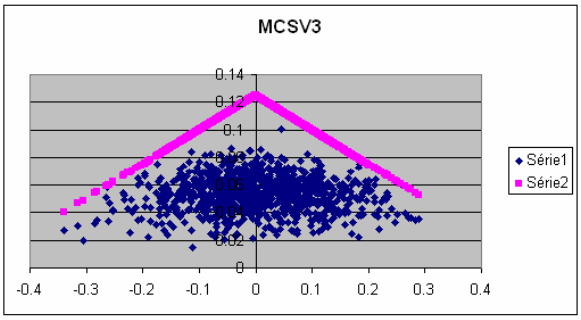

Figure 20: FC STZ simulated (8 links)

Settings: Cpk from 0.47 to 2

Findings:

- FC is satisfied,
- The STZ is not fully used.

With 4 links we reach FC satisfaction with Cpk from 0.666 to 3, with a good STZ filling:

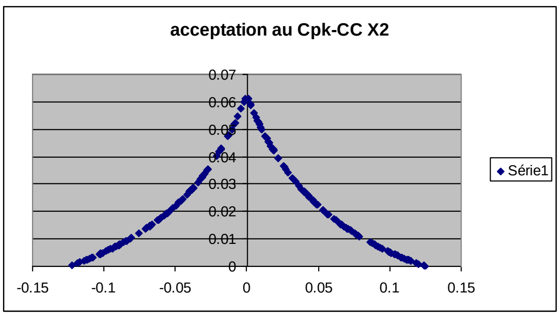

Figure 21: variable Cpk STZ (4links)

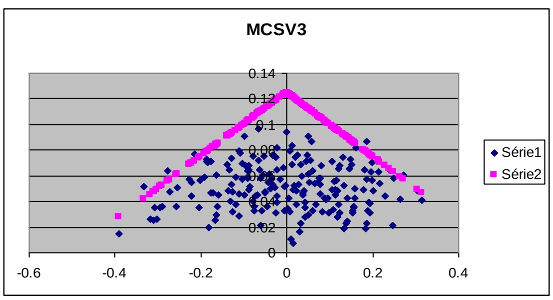

Figure 22 : FC STZ simulated (4 links), 200 run

6.2.3 *Improving the STZ efficiency:*

We expand ITx with a power ratio: 0.06

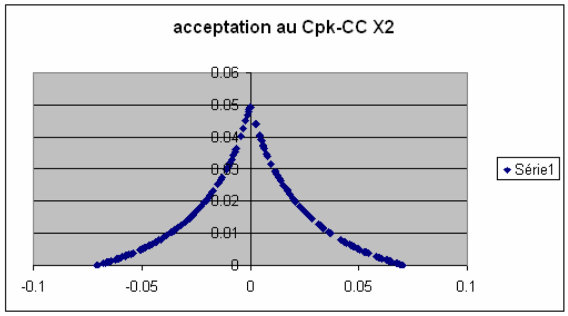

Figure 23 : variable Cpk STZ expanded

TRx is increased to +/-0.708; Cpk varies from 0.47 to 3, meaning a more concave curve.

FC quality is quite good:

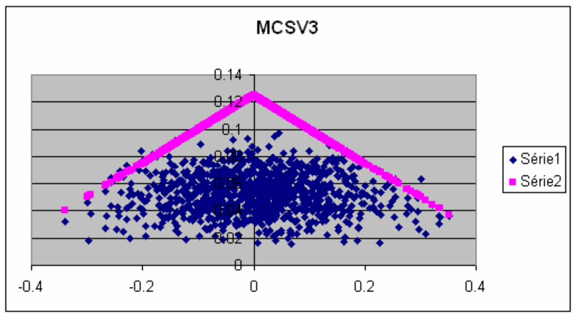

Figure 24: FC STZ simulated (8 links)

6.3 *Efficiency table*

The indicator chosen is the half-surface of the parameter STZ.

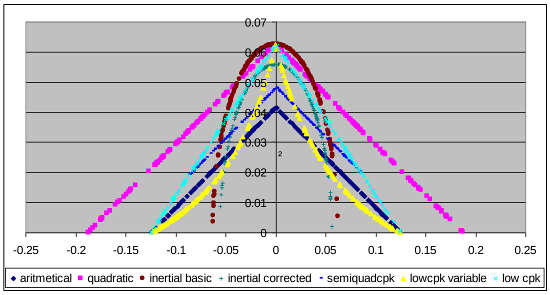

Figure 25: parameter STZ efficiency (4 links)

Arithmetical:	0.00260
Inertial regular:	0.00307
Inertial corrected:	0.00245
Semi-quadratic (0.6):	0.00291
“Low cpk” not optimized:	0.00390
“Low Cpk” variable:	#0.00263

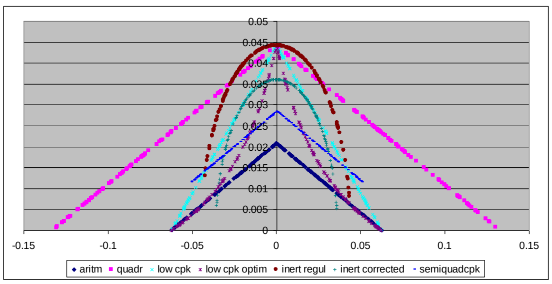

Figure 26: parameter STZ efficiency (8 links)

Findings:

- “Low Cpk variable” has poor efficiency results close from inertial corrected or semi-quadratic or even arithmetical for *n* low,
- “Low Cpk” is always more efficient than inertial corrected, but with a risk especially for *n* low,
- Deleting the external points is costly…
- Each tolerancing model has a trade-off between deviation and variance, regarding to the STZ areas that the model authorises or not.
- It is the only interest of TRx expansion, because the surface does not seem bigger, so what is saved in deviation is lost somewhere in variance.

7 Conclusions

We propose the following guide line:

1. If your quality requirement is not “zero non conforming Cpk” on the assembly[5], you can choose “inertial regular” or “Low Cpk” (or...Quadratic!)
2. Final choice is related to the deviation characteristics of your process producing the parts: for example if the drift is probable, rather choose “Low Cpk” (or Arithmetic) than Inertia (or Semi-quadratic). In other words, designer has to be supported by industrials (advanced manufacturing, quality, and suppliers) to set the model appropriated to the probable defaults.

remark: never forget that the above STZ models are “acceptation” models. They are supposed to act as filters on “production” models. The prerequisite before statistical tolerancing is to know these ones…The expected distribution law, combined with above acceptance law, will allow you relevant simulation and then NCR controlling.

For these reason, we do not think that highest priority is to go deeper into the statistics.

[5] Indeed the probability to get all Cpk or Inertia at the worst case value is very low. However it is advised to put this risk under control at least during components and process qualification, and if necessary by supply quality management.

Acknowledgments

This study is the follow up of the consequent work produced by SYMME laboratory in Annecy (France) passed years, and has to be considered as an attempt to understand and to apply their findings in the author's company. Thanks to INPG and G-Scop laboratory in Grenoble for giving me the opportunity of this publication and reviewing it

Appendix

Probabilistic tolerancing

The calculation of the FC_{TR} is done with assumption of an equiprobable distribution on the parameters. We can obtain FC_{TR}, by convolution (by digital processing) or with Gaussian assumption when *n* is high.(indeed $sigma_x = TR_x/\sqrt{(12)}$).
With this assumption we get TRx= 0.0765 for *n*=8
As for arithmetic, we have to define an acceptance rule, that will usually be Cpk=1.00

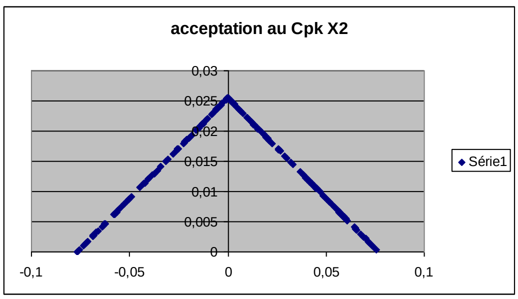

Figure 27: equiprobable STZ (8 links)

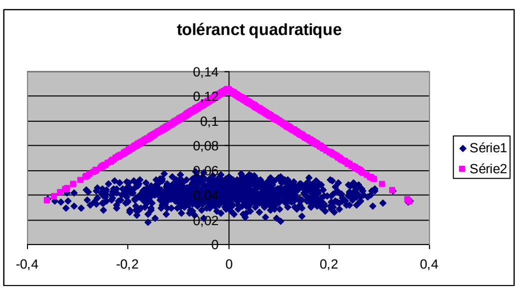

Figure 28: equiprobable FCSTZ (8 links)

FC is satisfied. It is possible to decrease slightly the Cpk target in order to allow more variance.

Quadratic tolerancing with XP E 04 008 acceptance rule

TRx unchanged, the deviation is limited to one sigma

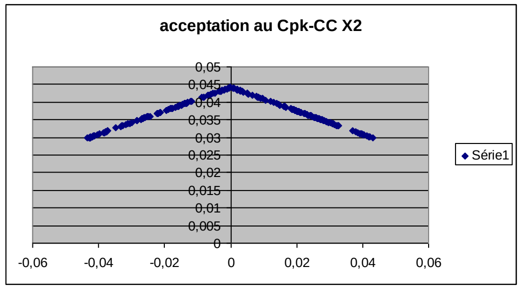

Figure 29: quadratic XP STZ (8 links)

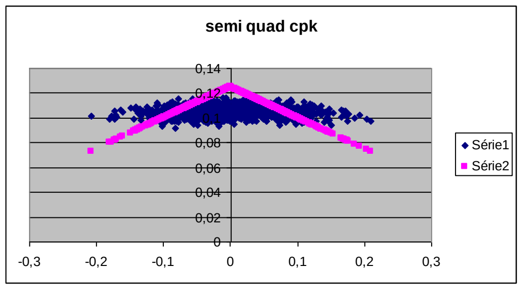

Figure 30 : FC STZ simulation for quadratic XP

Deviation is strongly limited, variance is still high

Synthesis with probabilistic and quadratic-XP:

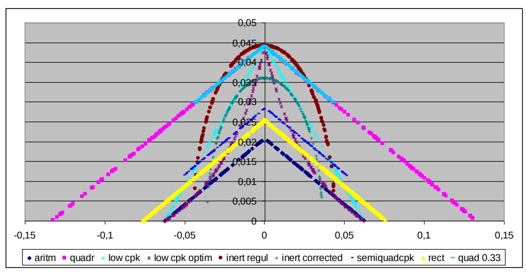

Figure 31 : FC STZ simulation yellow&blue

For *n*=8 the authorized deviation is quite identical with quadratic according XP acceptance rule or semi-quadratic, but efficiency& risk compromize is better with "low cpk" method.

If we attach importance to deviation, probabilistic method is interesting.

A compromise “low cpk” - “probabilistic” is interesting to investigate:

Trial with equiprobable TR; cpk from .7 to 2

For 8 links:

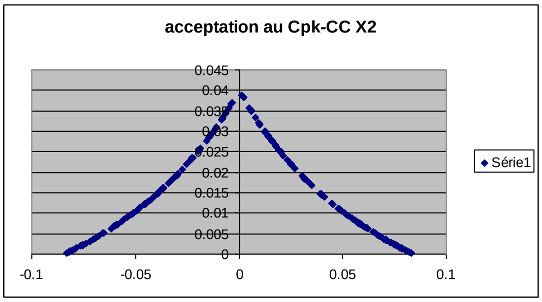

Figure 32: optimized equiprobable FTZ

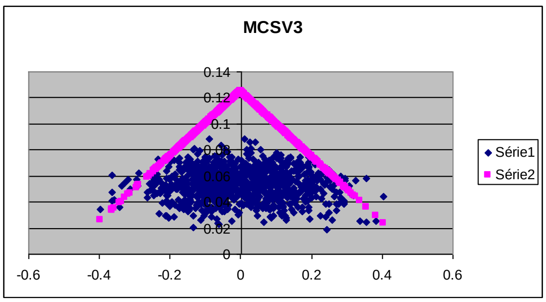

Figure 33 : FC STZ simulation for optimized equip.

Non conforming FC: 0.025

Synthesis for 8 links with new models:

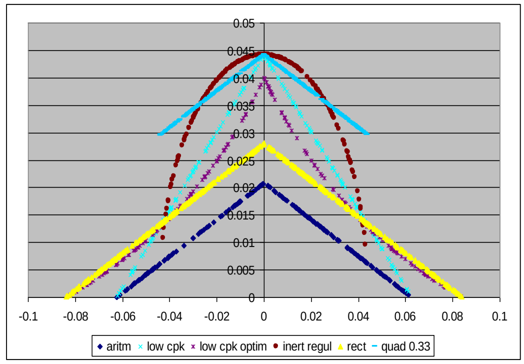

Figure 34: parameter STZ efficiency systhesis (8)

Optimized equiprobable in purple: we gain deviation but loose spread compared to “low cpk”. For 4 links:

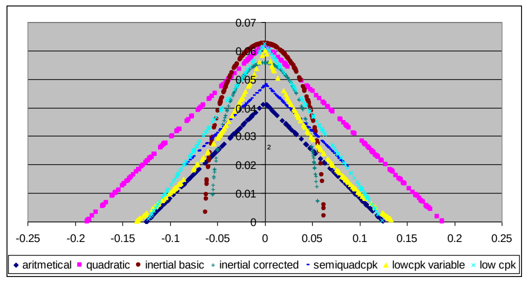

Figure 35: parameter STZ efficiency synthesis (4)

In yellow, “optimised equiprobable” Very close from previous low cpk model optimised.

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