Research Article

Passive Controller Design of Mass Exchanger Network

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Abstract
This work mainly focuses on applying the passivity concept to a mass exchanger network. Development of the mass exchanger state space model with controller design is proposed. The state space equations of the mass exchanger model have been developed based on assumptions a of lumped parameter system, including well mixing rich and lean streams, a linear equilibrium relation over the operating range and isothermal and isobaric conditions. The mass exchanger model is characterized as a non-passive system due to its positive passivity index, and according a weighting function is introduced to shift the system to passive region along with the controller tuning. A case study of a five-streams mass exchanger network with four control loops is demonstrated using a weihing function to again counter the non-passive characterstic. Multi-loop passive PI controllers of the mass exchanger network are developed. This passive control configuration is tested by both disturbance rejection and set-point tracking. The designed passive controllers are able to completely eliminate the impact of disturbance effects, and all control loops show good performance to track 4, 15, 3 and 1.5% change of the set-points.

Keywords: Mass Exchanger Network, State space model, Passivity concept, Multi-loop controller

1 Introduction

In most process industries, a Mass Exchanger Network (MEN) plays a crucial role in economic and environmental considerations. Mass exchanger networks are widely acceptable in many applications, such as purification and product recovery. Generally, the mass exchanger network consists of a number of mass exchangers, each of which is a direct-contact mass transfer unit that employs a Mass Separating Agent (MSA) in a lean stream to remove certain components from a rich stream. Mass Exchanger Networks (MEN) were first introduced by El-Halwagi and Manousiouthakis [1], [2]. Their work resulted in targets for the minimum amount of MSA and final configuration of the MEN that satisfied the assigned exchange duty at a minimum venture cost. Recently, a disturbance rejection model for mass exchanger network with recycle has been introduced [3]. The model was extended from a disturbance propagation model by Yang, *et al*. [4]. The structure

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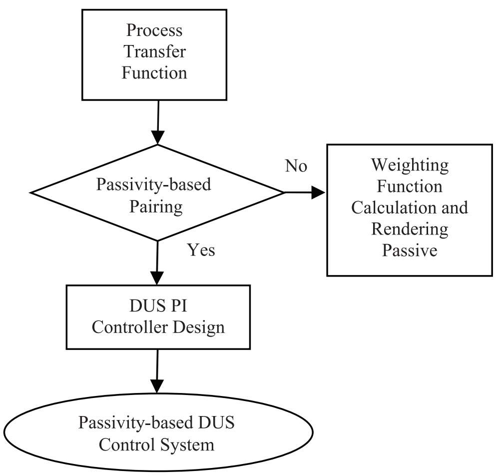

Figure 1: Passive-based decentralized controller synthesis procedure.

of a mass exchanger network with recycle is investigated regarding the optimal placement of recycle for a disturbance rejection problem. Consequently, a control point of view of MEN becomes a main motivation in this work. A way is that commonly used to describe the dynamics behavior of the system is state space, which is adopted throughout this work. In addition, the passivity theory, one of the techniques to analyze the stability of general processes, is implemented in the controller design stage

Through a lag of dynamic behavior of MEN and benefit of passivity theorem, control point of view of MEN and application of passivity concept are motivation of this work. The generic state space equations of a single mass exchanger with recycle are proposed through this work and then extended to a mass exchanger network. The selected structure of a mass exchanger network is used as a case study to illustrate a systematic passive controller design.

2 Systematic Synthesis of Passivity-based Decentralized Control and Stability

The passivity concept was systematically introduced to obtain a passivity-based decentralized unconditional stability control system as illustrated in Figure 1. Starting from process modeling, the target of this is to represent the model of the system and leads to its transfer function. Basically, an arbitrary pairing can be designed from its transfer function. Therefore,

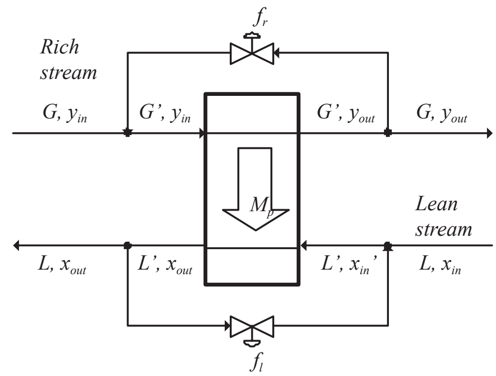

Figure 2: Sketch of a general mass exchanger unit with recycles.

an application of passivity theorem is introduced: the non-passivity can be driven to passive region by introducing weighting. After the system shows its passive property, the introduction of Decentralized Unconditional Stability (DUS) for PI controller design is implemented, and all of the parameters are calculated. In practice, alternative control structures can be addressed. Therefore, the passivity index needs to be considered to nominate the best system. The lower the passivity index, the more suitable control systems can be designed.

3 Generic State Space Equations of a Single Mass Exchanger with Recycle

A single mass exchanger with recycle is shown in Figure 2. In general, the duty of a Mass EXchanger unit (MEX) is to transfer the key component from a rich stream to a lean stream, leading to the composition decreasing in the rich stream from y_{in}' to y_{out} and increasing in the lean stream from x_{in}' to x_{out}. The dynamic model of a single mass exchanger is developed based on the controlled (outlet composition of rich and lean streams), manipulated (recycle fraction) and disturbance (inlet composition of rich and lean streams) variables of the system and the following assumptions:

- A mass exchanger model is an approximate lumped parameter system.
- Both rich and lean streams are well mixed.
- The equilibrium relation is linear over the operating range and can be expressed as $y = mx + c$
- Masses of each of the rich and lean streams in the mass exchanger are constant.

- The mass exchange operates under isothermal and isobaric conditions.

The linear dynamic models of the mass exchanger unit using an arithmetic mean of the overall mass transfer coefficient are presented as the following:

$$\frac{dy_{out}}{dt}=\frac{G-KA(1-\overline{f}_r)}{2aM_G}y_{in}+\frac{-G-KA(1+\overline{f}_r)}{2aM_G}y_{out}+\frac{KA(1-\overline{f}_r)}{2aM_L}x_{in}+\frac{KA(1+\overline{f}_r)}{2M_G}x_{out}+\frac{KA(\overline{y}_{in}-\overline{y}_{out})}{2aM_G}f_r+\frac{KA(\overline{x}_{out}-\overline{x}_{in})}{2M_G}f_l \quad (1)$$

$$\frac{dx_{out}}{dt}=\frac{L-KA(1-\overline{f}_l)}{2M_L}x_{in}+\frac{-L-KA(1+\overline{f}_l)}{2M_L}x_{out}+\frac{KA(1-\overline{f}_r)}{2aM_L}y_{in}+\frac{KA(1+\overline{f}_r)}{2aM_L}y_{out}+\frac{KA(\overline{y}_{out}-\overline{y}_{in})}{2aM_L}f_r+\frac{KA(\overline{x}_{in}-\overline{x}_{out})}{2M_L}f_l \quad (2)$$

To study the transient response of the mass exchanger unit by the passivity concept, the linear dynamic model of the mass exchanger is first represented in the state space form. Equations (1), (2) are arranged in the form of state space in order to find out the transfer function of the system as shown in Equation (3), (4).

$$\begin{bmatrix}\dot{y}_{out}\\ \dot{x}_{out}\end{bmatrix}=\begin{bmatrix}A_{11}&A_{12}\\ A_{21}&A_{22}\end{bmatrix}\begin{bmatrix}y_{out}\\ x_{out}\end{bmatrix}+\begin{bmatrix}B_{11}&B_{12}\\ B_{21}&B_{22}\end{bmatrix}\begin{bmatrix}f_r\\ f_l\end{bmatrix}+\begin{bmatrix}E_{11}&E_{12}\\ E_{21}&E_{22}\end{bmatrix}\begin{bmatrix}y_{in}\\ x_{in}\end{bmatrix} \quad (3)$$

$$\begin{bmatrix}y_{out}\\ x_{out}\end{bmatrix}=\begin{bmatrix}1&0\\ 0&1\end{bmatrix}\begin{bmatrix}y_{out}\\ x_{out}\end{bmatrix} \quad (4)$$

Where are the partitioned matrices shown in Equation (5)–(16).

$$A_{11}=\frac{-G-KA(1+\overline{f}_r)}{2aM_G} \quad (5) \qquad A_{12}=\frac{KA(1+\overline{f}_l)}{2M_G} \quad (6)$$

$$A_{21}=\frac{KA(1+\overline{f}_r)}{2aM_L} \quad (7) \qquad A_{22}=\frac{-L-KA(1+\overline{f}_l)}{2M_L} \quad (8)$$

$$B_{11}=\frac{KA(\overline{y}_{in}-\overline{y}_{out})}{2aM_G} \quad (9) \qquad B_{12}=\frac{KA(\overline{x}_{out}-\overline{x}_{in})}{2M_G} \quad (10)$$

$$B_{21}=\frac{KA(\overline{y}_{out}-\overline{y}_{in})}{2aM_L} \quad (11) \qquad B_{22}=\frac{KA(\overline{x}_{in}-\overline{x}_{out})}{2M_L} \quad (12)$$

$$E_{11}=\frac{G-KA(1-\overline{f}_r)}{2aM_G} \quad (13) \qquad E_{12}=\frac{KA(1-\overline{f}_l)}{2M_G} \quad (14)$$

$$E_{21}=\frac{KA(1-\overline{f}_r)}{2aM_L} \quad (15) \qquad E_{22}=\frac{L-KA(1-\overline{f}_l)}{2M_L} \quad (16)$$

4 Passivity Analysis of a Single Mass Exchanger Unit

A single mass exchange unit used for copper recovery in an etching plant [2] is used as an example for verify the model and apply the passivity concept. The numerical values of various parameters at steady state are shown in Table 1.

Table 1: Steady state information of a mass exchanger in a copper recovery unit

Parameter	Unit	Value
Rich stream mass rate	kg/s	0.1
Lean stream mass rate	kg/s	0.0925
Inlet rich stream composition	-	0.06
Inlet lean stream composition	-	0.03
Output rich stream composition	-	0.02307
Output lean stream composition	-	0.07
Rich stream mass accumulation	kg	50*
Lean stream mass accumulation	kg	50*
Recycle fraction	-	0
Overall mass transfer coefficient	kg/s	0.7079**

Remark * = hypothesis value
** = calculated value based on a steady state model

After substituting the numerical values at steady state into the matrix A, B, and E the state space of a single mass exchanger is determined. Thus, the process transfer function is shown in Equation (17).

$$\begin{bmatrix}\dot{y}_{out}\\ \dot{x}_{out}\end{bmatrix}=\begin{bmatrix}\frac{0.0003565s+6.595\times10^{-7}}{s^2+0.02057s+3.57\times10^{-5}} & \frac{0.0002829s+5.234\times10^{-7}}{s^2+0.02057s+3.57\times10^{-5}}\\ \frac{-0.0003565s-7.13\times10^{-7}}{s^2+0.02057s+3.57\times10^{-5}} & \frac{-0.0002829s+5.234\times10^{-7}}{s^2+0.02057s+3.57\times10^{-5}}\end{bmatrix}\begin{bmatrix}f_r\\ f_l\end{bmatrix} \quad (17)$$

The dynamic response of the outlet composition of the rich and lean streams of copper recovery unit shows that the composition of the rich stream and

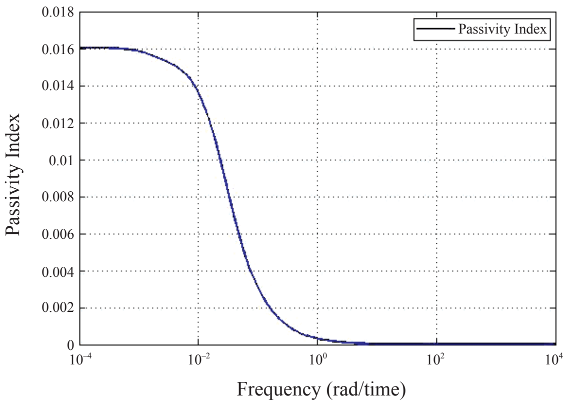

Figure 3: Passivity index of copper recovery unit.

that of the lean stream, are initially at 0.060 and 0.030; respectively. Finally, their concentrations go to 0.023 and 0.070, values which are equal to steady-state concentrations of the mass exchanger proposed by El-Halwagi and Manousiouthakis [2]. Thus, the developed dynamic model of the mass exchanger unit is verified and shown to be stable in its response after the passivity concept is applied. The passivity index of the mass exchanger unit is shown in Figure 3.

From this figure, it can be seen that the passivity index of this system is positive along the frequency range 10^{-4}–10^{4} rad/hr; therefore, this system is non-passive. Up to this point, it is clear that the mass exchanger is non-passive.

5 State Space Equations of a Mass Exchanger Network

Consider a five-streams mass exchanger network problem which was investigated by El-Halwagi and Manousiouthakis [1] and further studied by Yan and Huang [3] from the disturbance rejection point view, suggesting that the most suitable control structure is that shown in Figure 4. Table 2 lists the design data, this is the normal operating information and target concentration of the streams. The control loop connections, also presented in figure 4, work as follows. At normal operation, all valves are half open and the pinch point is located at the concentration of 0.05 for rich streams *R1* and *R2*, and 0.05, 0.09, and 0.024 for lean streams *L1*, *L2*, and *L3*, respectively. The equilibrium relations for a key component between a rich stream and each lean stream are given as shown in Equation (18)–(20).

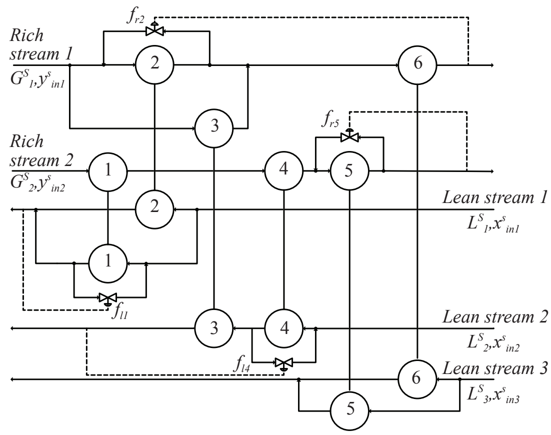

Figure 4: Grid diagram of five-stream mass exchanger network with control loop.

$$y = 0.8x_1 + 0.002 \tag{18}$$

$$y = 0.5x_2 \tag{19}$$

$$y = 0.2x_3 \tag{20}$$

Table 2: Stream data of the five-stream mass exchanger network

Source Stream	Mass Rate (kg/s)	Source Composition	Target Composition
G1	1.3	0.115	0.025
G2	1.5	0.100	0.025
L1	2.5	0.050	0.110
L2	0.5	0.035	0.109
L3	0.6	0.010	n/a

The linear dynamic models of the mass exchanger network [5] are derived in the same manner as those of single mass exchanger unit. Therefore, the dynamic model of the mass exchanger network as a form of state space equation can be derived with the dimensions of matrices A, B, C, D and E being 12×12, 4×12, 12×4, 4×4 and 5×12, respectively.

From all of the above, the output stream composition can be determined. The compositions of rich streams 1 and 2 and lean streams 1 and 2 start at initial point 0.11, 0.1, 0.05 and 0.035, receptivity and finally reach values of steady concentration, 0.025, 0.025, 0.11 and 0.105, respectively results which agree with results of Yan and Huang [3]. It can be concluded that, this model gives good agreement with the literature.

6 Passive Controller Design of Mass Exchanger Network

Using the state space equation of the mass exchanger network, the passivity concept can be applied to the system by firstly determining the passivity index. The passivity index of five-stream mass exchanger network is possitive along the frequency range 10^{-4}–10^{4} rad/hr [6]–[8]; therefore, this system is non-passive. To shift the system from being a non-passive to being a passive system [9], a stable minimum phase-transfer function, called a weighting function w(s), is added into this system. Consequently, the weighting function for this network model is shown in Equation (21), and its passivity index is depicted in Figure 5.

$$w(s) = \frac{0.0027s(s+0.4393)}{(s+0.001)(s+0.001)} \tag{21}$$

After the system has the absorbed weighting function applied to it, the system shows that it is passive insofar as there is obtained a negative value of the passivity index, as shown in Figure 5. However, the weighting function cannot be directly put into the system, but weighting function can be absorbed into the controller in the passive controller design step [8].

Since the system is non-passive, the weighting function should be absorbed into the controller and the design equation of controller is modified as shown in Equation (22). Therefore, the passive controllers of each loop control are determined as shown in Equation (23)–(26).

$$k_i^{'}(s) = k_i^{+}(s)[1-w(s)k_i^{+}(s)]^{-1} \tag{22}$$

$$k_1^{'}(s) = \frac{(s+0.1)(s+0.001)(s+0.001)}{s(s+0.0019)(s+0.0001)} \tag{23}$$

$$k_2^{'}(s) = \frac{(s+0.1)(s+0.001)(s+0.001)}{s(s+0.0018)(s+0.0002)} \tag{24}$$

$$k_3^{'}(s) = \frac{(s+0.1)(s+0.001)(s+0.001)}{s(s+0.0013)(s+0.0007)} \tag{25}$$

$$k_4^{'}(s) = \frac{(s+0.1)(s+0.001)(s+0.001)}{s(s+0.0019)(s+0.0001)} \tag{26}$$

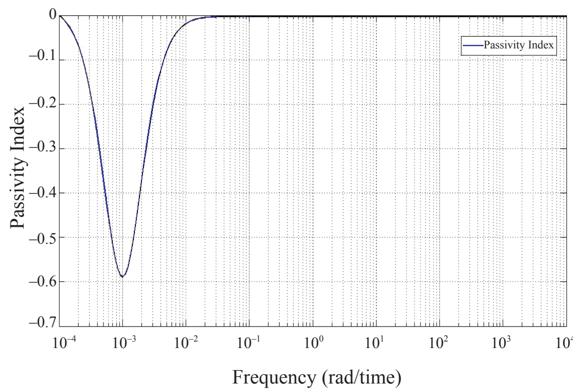

Figure 5: Passivity index of the five-stream mass exchanger network after the introduction of the weighting function.

7 Dynamic Study of the Mass Exchanger Network

Open-loop and closed-loop models of the mass exchanger network, depicted in Figure 6, are developed by MATLAB-simulink simulation. The state space equation as discussed in section 5 is used to describe the dynamic behavior of the system. In addition, passive controllers, as designed in section 6, are also incorporated in closed-loop model. The system has four control loops in which the output composition of lean stream 1, rich stream 1, lean stream 2 and rich stream 2 are controlled using manipulated lean stream recycle valve of MEX no.1, rich stream recycle valve of MEX no.2, lean stream recycle valve of MEX no.4 and rich stream recycle valve of MEX no.5, respectively. Moreover, compositions of lean stream 1, rich stream 1, lean stream 2 and rich stream 2 are defined as disturbances.

The designed passive controllers are tested for both disturbance rejection and set-point tracking problems. The input compositions of lean stream 1, rich stream 1, lean stream 2 and rich stream 2 experience most fluctuate as 0.0025, 0.0080, 0.0040 and 0.0060; respectively.

The open-loop model is tested with both a single step disturbance in each loop and simultaneous disturbances in the load. As a result, off-spec composition in the output stream is as shown in Table 3. Lean stream 1 is the first to contact with the upstream mass exchanger, thus the step change in composition of input lean stream 1 results in the changing of other output concentrations, as shown in case no. 1.

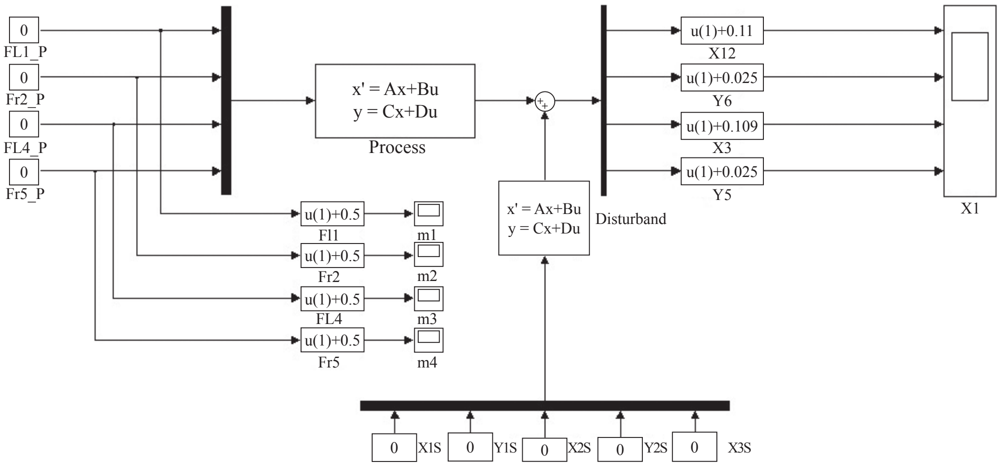

Figure 6a: Matlab-simulink model for the four main streams of the five-stream mass exchanger network problem Open-loop model.

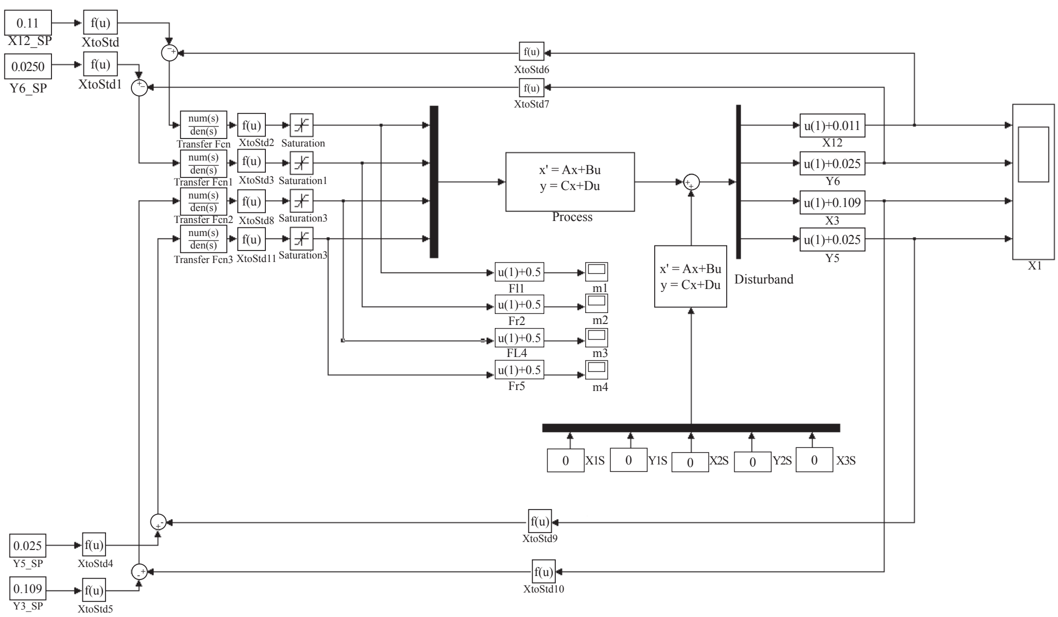

Figure 6b: Matlab-simulink model for the four main streams of the five-stream mass exchanger network problem closed-loop model.

Table 3: Composition deviation for step change of disturbance

Case	Deviation from Target Composition			
	Lean Stream 1	**Rich Stream 1**	**Lean Stream 2**	**Rich Stream 2**
1. Step change lean stream composition 1	0.001	0.001	0.006	0.001
2. Step change lean stream composition 2	0.000	0.000	0.005	0.001
3. Step change rich stream composition 1	0.002	0.001	0.006	0.000
4. Step change rich stream composition 2	0.003	0.000	0.006	0.001
5. Step change 4 streams composition simultaneous	0.007	0.002	0.009	0.001

In to case no. 2, lean stream 2 undergoes a step stepped change.This gives results as follows. There is no deviation in the output of lean stream 1 because lean stream 2 is a secondary lean stream that employs to the network after lean stream 1 takes its duty. The step change in input lean stream 2 has a slight effect on rich stream 1, because there is less mass transfer load between lean stream 1 and rich stream 1. Lean stream 2 is the first to contact with MEX unit 4, which is the mass exchanger unit between lean stream 2 and rich stream 2, as a result of a major concentration deviation of output lean stream 2 and rich stream 2.

In addition, there is no mass exchanger connect between rich stream 1 and rich stream 2; therefore the step change in rich stream 1 only affects rich stream 1 and the other two lean streams, and likewise rich stream 2 only affects as shown in cases no. 3 and no. 4.

Output composition deviations occur after a disturbance is loaded to the system as discuss above. Thus, passive controllers, as designed in section 6, take a role in disturbance rejection. The closed-loop output composition and control valve response for the case where in a single disturbance is applied to the system are shown in Figure 7. Poles of the controllers occur close to the origin, indicating that the controllers are sensitive. The result shows that all control variables reach their set-point without any offset very quickly by manipulating the control valve. Therefore, the designed passive controllers work well under the disturbance rejection problem.

For worst situation, all disturbances are applied to the system simultaneously as a result of deviation of output composition, as shown in case no. 5 (Table 3). The designed passive controller is applied to the system. The closed-loop responses of the output composition and recycle valve position are shown in Figure 8 and 9, respectively.

The results show that the valve of controller loop 1 (Lean stream recycle valve of MEX no. 1) is saturated, leading to off-spec composition of lean stream 1. Thus, the control structure should be redesigned to satisfy disturbance load, or split-range control should be considered.

Moreover, the designed passive controllers are tested by set-point tracking problem as well. Under no disturbances in the load to the system, the set-point of each control loop undergoes a step change. The control loops 1, 2, 3 and 4 are able capable to track 4%, 10%, 3% and 1.5% deviation of its set-point, respectively, as shown in Figure 10.

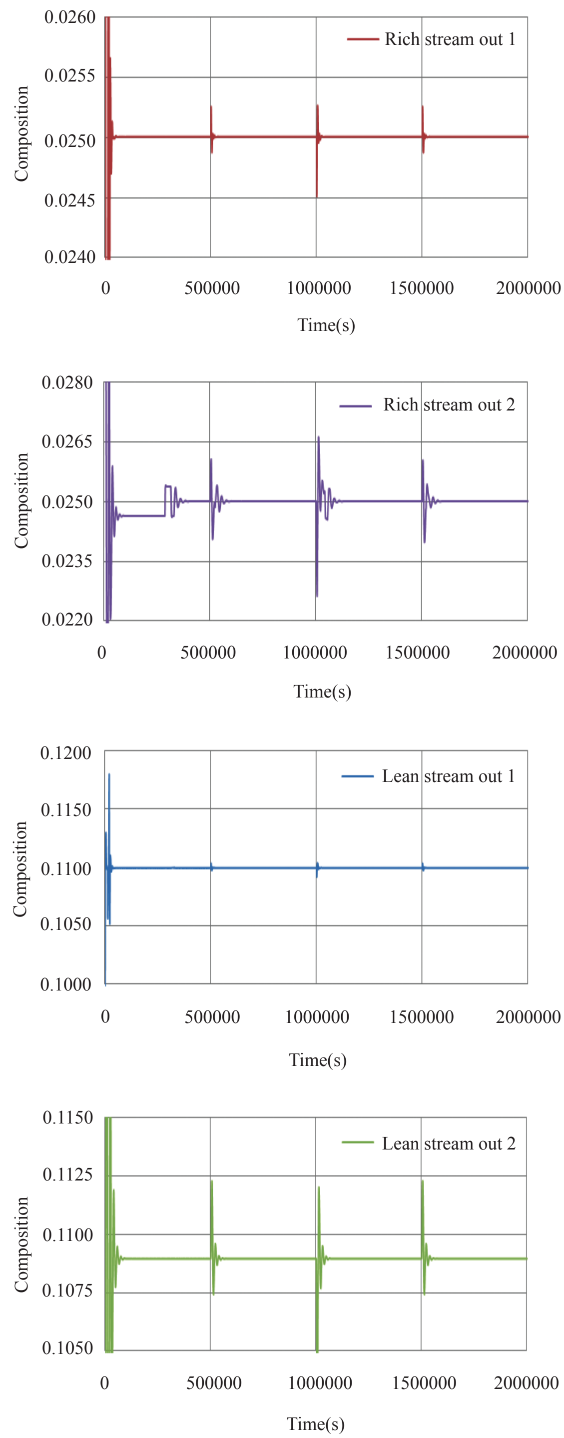

Figure 7a: Closed-loop response for a step change in disturbances from lean stream 1.

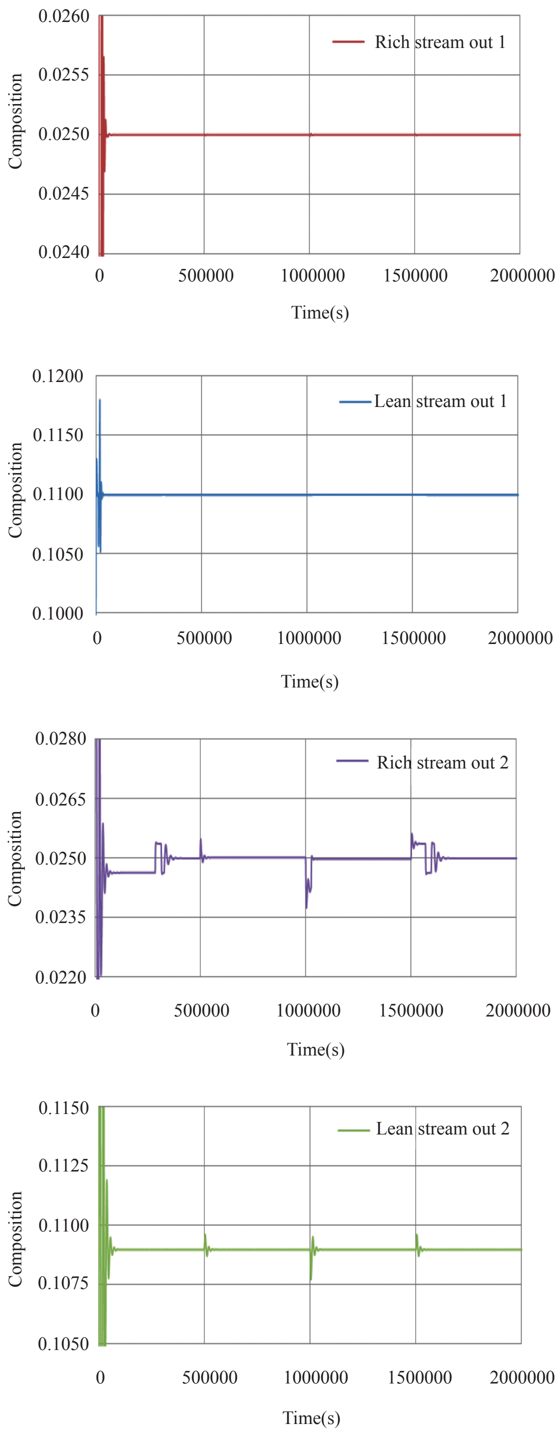

Figure 7b: Closed-loop response for a step change in disturbances from lean stream 2.

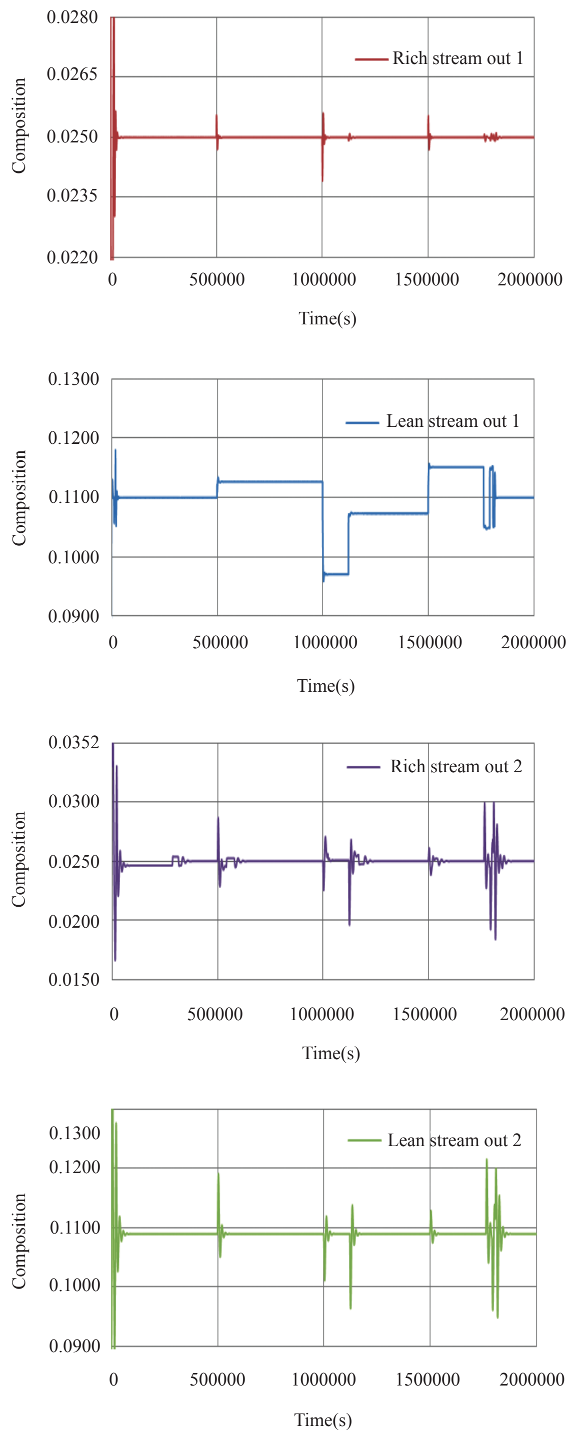

Figure 8: Closed-loop response after implementing step change to all input compositions simultaneously.

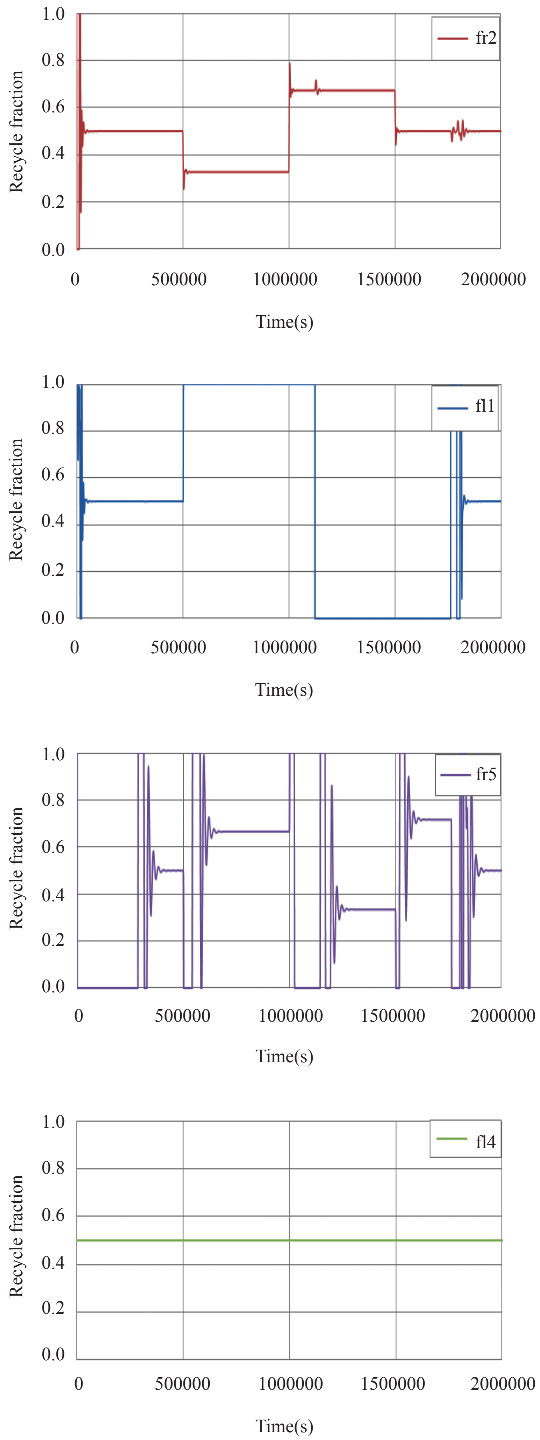

Figure 9: Closed-loop recycle valve position after implementing step change to all input compositions.

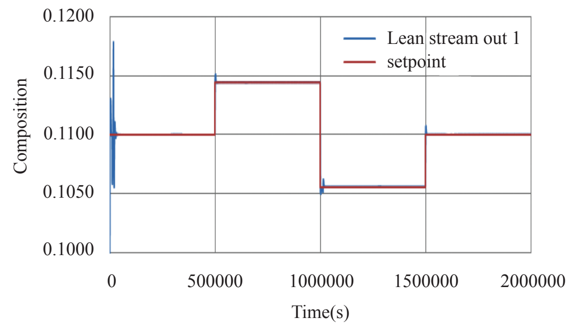

Figure 10a: Set-point tracking of controller loop 1.

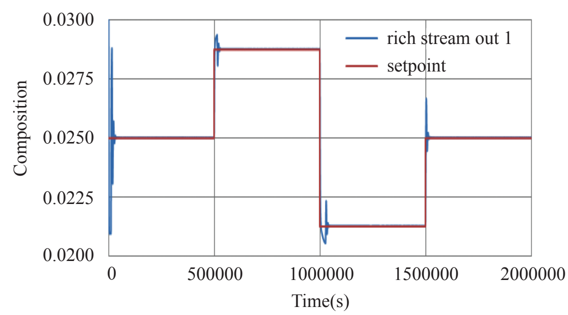

Figure 10b: Set-point tracking of controller loop 2.

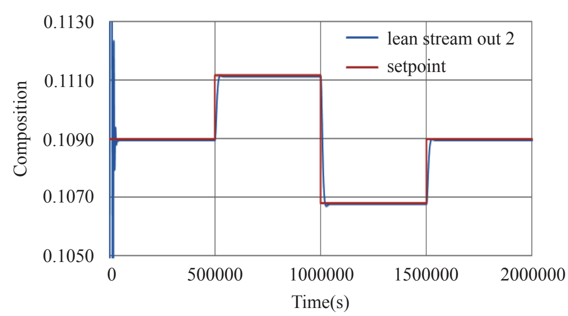

Figure 10c: Set-point tracking of controller loop 3.

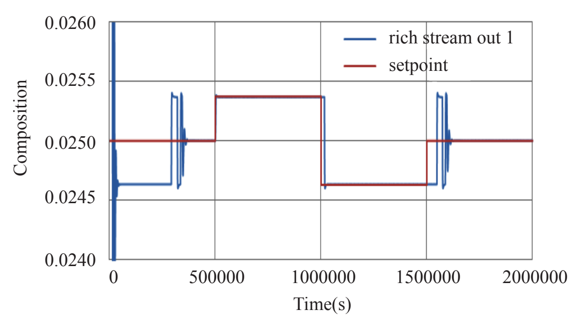

Figure 10d: Set-point tracking of controller loop 4.

8 Conclusion

This work has mainly focused on the development of state space equations for a mass exchanger network to which can be applied passivity concept. The mass exchanger model has been developed based on assumptions of an approximate lumped parameter system, of both rich and lean stream being well mixed, of a linear equilibrium relation adding over the operating range and of operation under constant isothermal and isobaric conditions. After applying the passivity concept on a mass exchanger unit, it was found that the mass exchanger is characterized as a non-passive system due to its positive passivity index.

Consequently, the passivity concept was extended to a mass exchanger network. Following to Yan and Huang [3], a five-stream mass exchanger network is used as a case study to apply the passivity concept [8]. The mass exchanger network has four control loops in which where the output composition of lean stream 1, rich stream 1, lean stream 2 and rich stream 2 are controlled by manipulate lean stream recycle valve of MEX no.1, rich stream recycle valve of MEX no. 2, lean stream recycle valve of MEX no. 4 and rich stream recycle valve of MEX no. 5, respectively. Upon investigation, it is found that the system is non-passive, so a weighting function shown in Equation (27) and added to drive the system to because passive. Multi-loop passive PI controllers of the mass exchanger network are investigated for control loops 1, 2, l3 and 4 one after another.

$$w(s) = \frac{0.0027S(s+0.4393)}{(s+0.001)(s+0.001)} \tag{27}$$

The designed passive control of the five-stream mass exchanger network is tested for both disturbance rejection and set-point tracking problems. For disturbance rejection problem, the input compositions of lean stream 1, rich stream 1, lean stream 2 and rich stream 2 undergo step changes of 0.0025, 0.0080, 0.0040 and 0.0060 deviation respectively. In addition, the set-point tracking capability is investigated. The control loops 1, 2, 3, 4 are able to track 4, 15, 3 and 1.5% changes in the set-point.

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