Research Article

Model-robust G-optimal Designs in the Presence of Block Effects

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Abstract
Due to the uncertainty of possible reduced models prior to data collection, this paper considered using experimental designs that are robust across the set of potential models. In this study, blocking effects were combined into all possible models which were obtained from weak heredity principle (WH). The objective of this article was to propose the geometric mean of G-optimalities as a new alternative for finding robust response surface designs against model misspecification. The proposed criterion is so-called a weighted G-optimality criterion (G_w). The genetic algorithm (GA) was employed to optimize the weighted G-optimality criterion for finding designs. Robust designs having 2 and 3 design variables in hypercube were generated with an appropriate number of design points in each blocks and the number of blocks in this study are 2, 3, and 4. The scheme for weighting the criteria was to give more weight to a model with a larger number of parameters. The resulting weighted G-optimal designs have higher G-efficiencies compared to those of G-optimal designs if a true model is the first-order or interaction models. The G-efficiency of a weighted G-optimal design is slightly less than that of a G-optimal design even when the true model is a second-order model. Furthermore, design points identified from the GA are also presented, which would be very useful in practice for those intending to implement an experimental design for data collection.

Keywords: Experimental designs, Response surface designs, Weighted G-optimality, Weak heredity, Genetic algorithm

1 Introduction

Response surface designs are one class of experimental designs that are important for developing, improving and optimizing the performance of an industrial process. It is generally affiliated with the approximation of unknown complicated function by using a lower-order polynomial model, usually first-order, interaction, or second-order model. In situations in which data for every combination of factor levels cannot be collected under identical conditions, blocks should be formed to reduce variability. A small exact response surface design is commonly constructed by assuming a second-order model. The model for k design variables and b blocks can be expressed as Equation (1):

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \beta_{ij} x_i x_j + \sum_{l=1}^{b-1} \delta_l + \varepsilon \quad (1)$$

where $x_1, x_2, \ldots, x_k$ are the k design variables, y is an observed response, the β's are the parameter

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coefficients to be estimated, δ_1 is the *lth* block effect and ε is a normally distributed error term having an expectation of zero and variance σ^2.

There are many designs choices for a considered model. Selecting a good design is very important, and there are many design criteria that could be used for selection. Design optimality criteria are generally concerned with the optimal properties of the X^TX matrix, where X is the model matrix [1].

In this paper, we focus on the G-optimality criterion, which centers on minimizing the prediction variance. G-optimality minimizes the maximum of the Scaled Prediction Variance (SPV) function. This criterion was first proposed by Smith [2]. G comes from the word "Global" as the SPV is calculated with all points in design space χ, not just design points. Let ξ* be a G -optimal design, such that [Equation (2)]

$$\begin{aligned}\xi^* &= \arg\min_{\xi\in\Xi}\max_{x\in\chi} \text{SPV(x)} \\ &= \arg\min_{\xi\in\Xi}\max_{x\in\chi} N\,\text{Var}(\hat{y}(x)) \\ &= \arg\min_{\xi\in\Xi}\max_{x\in\chi} N\, {x^{(m)}}^{\text{T}} (\text{X}^{\text{T}}\text{X})^{-1} x^{(m)}, \qquad (2)\end{aligned}$$

where Ξ is the set of all possible exact designs on design space χ,
$x^{(m)} = [1, x_1, ..., x_k, x_1^2, ..., x_k^2, x_1x_2, ..., x_{k-1}x_k]$,
and N is the design size. The common efficiency measure for any proposed design and corresponding model based on G-optimality is called the G-efficiency [Equation (3)]:

$$\text{G-efficiency} = \frac{p}{\max_{x\in\chi} N\, {x^{(m)}}^{\text{T}} (\text{X}^{\text{T}}\text{X})^{-1} x^{(m)}} \times 100\%, \qquad (3)$$

where p is the number of model parameters. For more details on optimal design, see Atkinson *et al*. [3].

Chipman [4] suggested two classes of reduced models including the weak heredity (WH) and strong heredity (SH) principles. A model can be inferred with vector Δ containing '1' and '0', where '1' represents the corresponding term included in the model and '0' represents the term not included in the model. The symbols Δ_i, Δ_{ii} and Δ_{ij} represent the indicator function values for the *ith* first-order effect, the *iith* second-order effect, and the *ijth* interaction effect, respectively. Weak heredity (WH) requires that either the $\beta_i x_i$ or $\beta_j x_j$ term (or both) is contained in the model if the $\beta_{ij} x_i x_j$ term is included in the model, and requires the $\beta_i x_i$ term must be in the model if the $\beta_{ii} x_i^2$ term is included in the model. For k = 2, $\Delta = (\Delta_0, \Delta_1, \Delta_2, \Delta_{11}, \Delta_{22}, \Delta_{12})$ represents the corresponding Δ vectors for 6 parameters in the second-order model (without blocks). There are 17 WH reduced models. For k = 3, the second-order model consisting of 10 parameters (without blocks) will give 185 WH reduced models.

Chomtee and Borkowski [5] considered the design with 3 variables in a spherical design region over sets of reduced models based on weak and strong heredity and presented weighted D-, A-, G-, and IV-optimality criteria using prior probability assignments to model effects. Chairojwattana, Borkowski, and Chaimongkol [6] developed a genetic algorithm for generating designs that optimize the weighted D- and G-optimality criteria for second-order response surface designs; the weighted average of the efficiency values across all models are based on the arithmetic mean while the weights are determined by prior probability assignments to model effects. Limmun *et al*. [7] generated weighted A-optimality criterion and Limmun *et al*. [8] generated weighted IV-optimal for mixture designs using arithmetic mean as a criterion. In literature, there is no paper using the geometric mean as a criterion.

In this research, the weighted G-criterion (G_w) based on the geometric mean is used to construct designs. The objective of weighted G-optimality (G_w) is to maximize the weighted average of G-efficiencies in the design region over a set of reduced models. Thus, a weighted G-optimal is considered a robust design against a set of multiple possible models. The scope in this study involves the construction of optimal response surface designs over the sets of reduced models based on the weak heredity principles of a second-order model with 2, 3, and 4 blocks for 2 and 3 design variables. The number of design points starts from 8 to 18. The resulting robust designs obtained from weighting all reduced models will be compared with the optimal designs.

2 Using Weighted G-optimality to Construct Robust Designs

Let M be the number of reduced models where the initial or "full" model is the second-order model with blocks given in Equation (1). We define a set of model weights such $\{w_1, w_2, ..., w_M\}$ that $\sum_{i=1}^{M} w_i = 1$. The weights are applied for calculating the weighted G-optimality criterion (G_w). For model i, a weight $w_i = \frac{p(i)}{\mathrm{D} \times m(p(i))}$ is assigned, such that $p(i)$ equals the number of non-block parameters in model i, $m(p(i))$ is the number of models having $p(i)$ non-block parameters, and $\mathrm{D} = \sum_{p=1}^{t} p$ with $t = \binom{k+2}{2}$. The following method is used for k = 2 and 3 variables. For example, the second-order model without blocks has 6 parameters for k = 2, therefore, $\mathrm{D} = \sum_{p=1}^{6} p = 21$. That is, models having more parameters will obtain more weight. Here, the geometric mean is newly proposed for calculating the G_w -optimality criterion.

Let Ξ be the set of all possible exact designs on design space χ, the G_w -optimality criterion seeks a design ξ^* satisfying [Equation (4)]

$$\xi^* = \arg\min_{\xi \in \Xi} \left(\max_{a_i \in \chi} \prod_{i=1}^{M} \left[a_i^{(m)\mathrm{T}} \mathrm{M}_i^{-1}(\xi) a_i^{(m)} \right]^{w_i} \right), \qquad (4)$$

where $\mathrm{M}_i = \mathrm{X}_{(i)}^{\mathrm{T}} \mathrm{X}_{(i)} / N$ is a moment matrix for model i and N is the design size, $a_i^{(m)}$ is an expanded vector corresponding to the terms in the model. Thus, the corresponding G-efficiency is defined as [Equation (5)]

$$\mathrm{G}_w = \prod_{i=1}^{M} \mathrm{G}_i^{w_i}, \qquad (5)$$

where $\mathrm{G}_i = \frac{p}{\max_{a_i \in \chi} N \, a_i^{(m)\mathrm{T}} (\mathrm{X}^{\mathrm{T}}\mathrm{X})^{-1} a_i^{(m)}} \times 100\%$,

G_i is called the G-efficiency of the *ith* reduced model. The use of geometric mean is considered because the design should be robust to model reduction and should be able to fit all parameters for reduced models. While the weighted optimality criterion based on the arithmetic mean does not guarantee that all reduced models can be fitted. This is inconsistent with the goal of finding a model-robust design.

3 Genetic Algorithms

Genetic algorithms (GAs), computer-based strategies for searching and developing solutions to problems were first described by Holland [9]. The GA is taken from biological population genetics and the tenet of natural selection. This algorithm is based on the survival of the fittest biological essential; individuals modulate themselves to their environment and then evolve themselves into more desirable individuals. GAs have been applied to the generation of optimal response surface designs [10].

Borkowski [11] developed a GA to generate near-optimal D, A, G, and IV small exact N-point designs for second-order models in the hypercube. Thongsook *et al*. [12] presented a GA to generate–optimal designs for conditioned mixture regions when quadratic terms are of primary interest. Limmun *et al*. [7] developed a GA to generate weighted A-optimality criterion for mixture designs, while Limmun *et al*. [8] used GA to generate weighted IV-optimal mixture designs. It was found that the GA-generated designs are robust across a set of potential mixture models. Mahachaichanakul and Srisuradetchai [13] used the GA to construct D- and G-optimal robust designs against missing data. It is viewed that the GA is particularly attractive because it is relatively easy to achieve the objective function and can find good solutions in a reasonable amount of time. Unlike other optimization algorithms, the GA can be used for any objective function and is extremely flexible, making GAs highly useful in practice.

The GA will generate an exact N - point k - variable response surface design with a variety of blocking structures. A chromosome is an $N \times k$ matrix displaying the N design points in k factors. The goal is to find an $N \times k$ matrix that optimizes a design optimality criterion. A gene is defined as a row of chromosomes (design), and a genetic variable can be any design variable in a gene (or row). Let x_{ij} be the *jth* genetic design variable in row *ith* of a chromosome. The k-dimensional hypercube design region $[-1,1]^k$ directs a gene's possible values with each $x_{ij} \in [-1,1]$. An objective F function is a measurement function to measure a chromosome's fitness, which is a solution and the function we wish to optimize. F uses a chromosome as an input to produce the objective function value as the output, where larger objective function values are interpreted as having greater fitness.

3.1 *Initiation process*

The initiation of every generation will have a population that contains a fixed number M of chromosomes where M is odd.

3.2 *Selection process*

The best chromosome is selected after generating the initial population of M chromosomes. The best chromosome (elite chromosome) is the chromosome with the greatest objective function $F_G(X)$ value. This affects the next generation of chromosomes. To create the next generation of offspring chromosomes, randomly select $(M-1)/2$ pairs from the remaining $M-1$ non-elite chromosomes (parent chromosomes) before the reproduction process.

3.3 *Reproduction process*

Reproduction brings about the evolution in some characteristics of the chromosome to originate the next generation of chromosomes. After the reproduction process is done, we will obtain M–1 offspring chromosomes, which are related to the M–1 parent chromosomes. If the best offspring chromosomes have a higher objective function value than the value of the elite chromosome, the offspring chromosome with the highest objective function value will become a new elite chromosome. Therefore, the elite chromosome and the M–1 offspring become the future parents and survive to create the next generation of M chromosomes. The reproduction process can be changed according to the researcher and the nature of interesting solutions. However, it comprises the same idea as biological population genetics. For each operator, a probability test is performed on each row of A and B. Let A_a be the *ath* row of A and B_b be the *bth* row of B. A reproduction operator will be applied if a probability test is passed (PTIP). That is, for any reproduction operator (say g) and for a specified α_g, a PTIP if $0 \le u \le \alpha_g$ where $u \sim Uniform(0,1)$.

3.3.1 The swap rows (sr) gene operator

If a PTIP for row A_a of A occurs, the operator will exchange A_a with a random row B_b of B. The set of α_{sr} values is $0.002 \le \alpha_{sr} \le 0.02$.

3.3.2 The swap cut point (scp) gene operator

If a PTIP for row A_a of A occurs, the operator will exchange the last two decimal digits of the k genetic design variables of A_a with the last 2 decimal digits of the k genetic design variables for a random row B_b of B. The set of α_{scp} values is $0.005 \le \alpha_{scp} \le 0.02$.

3.3.3 The swap block (sb) gene operator

If a PTIP occurs for row j in block b (in either A or B), the operator will exchange row j in block b with a random row from another block. The remaining operators are applied to the genetic variables in the rows of either A or B. The set of α_{sb} values is $0.002 \le \alpha_{sb} \le 0.02$.

3.3.4 The swap coordinates (sc) gene operator

If a PTIP occurs for x_{ij} of A, the operator will exchange a x_{ij} of A with a random x_{kl} of B. The set of α_{sc} values is $0.002 \le \alpha_{sc} \le 0.02$.

3.3.5 The zero (z) gene operator

If a PTIP occurs for A_{ij}, then x_{ij} is changed to 0. The set of α_z values is $0.01 \le \alpha_z \le 0.05$.

3.3.6 The extreme (e) gene operator

If a PTIP occurs for x_{ij}, the x_{ij} is randomly set to either 1 or –1. The set of α_e values is $0.01 \le \alpha_e \le 0.10$.

3.3.7 The creep (c) operator

If a PTIP occurs for x_{ij}, then a random variate from $N(0, \sigma^2)$ is added to x_{ij} to create a new x^*_{ij}. The variance σ^2 is set by the researchers. The idea is to slowly change the value in each generation. If the creep operator takes $x^*_{ij} > 1$ or $x^*_{ij} < -1$, it will be set to 1 or –1, respectively. The set of α_c values is $0.025 \le \alpha_c \le 0.10$.

3.4 *Convergence checking*

If the objective function for the best chromosome in the new generation is not improved across many generations, then the GA will stop because no further improvement can be found.

4 Results and Discussion

For designs generated by GA, designs having k = 2 variables and b = 2 blocks are shown in Tables 1 and 2. The "AM" stands for the term "all models", which means the robust design that optimizes weighted G-optimality of all models. The resulting design points are in the last column. The "FM" stands for the term "full model only", which means the optimal design for only the second-order model, as in Equation (1).

The G_w -efficiencies referring to the weighted G-optimality efficiencies will be calculated for the resulting (robust) designs and the optimal designs (for the second-order model). Also, the G-optimality efficiencies are calculated for both robust and optimal designs. The G_w -efficiencies of the AM designs must be greater than those of the FM designs because the AM designs optimize the G_w giving weights for all reduced models. For example, for N = 8 and the block sizes are 4 and 4, the corresponding G_w - and G-efficiencies of the AM design are equal to 75.03 and 80.40, respectively. The design points in the 1st block are denoted by superscript 1, i.e. $(0.35, -1)^1$, $(-1,1)^1$, $(-0.78, 0.17)^1$, and $(1,1)^1$, while design points in the 2nd block are given as superscript 2, i.e. $(1,-1)^2$, $(-0.35,1)^2$, $(-1,-1)^2$, and $(0.78,0.15)^2$. The corresponding G_w - and G-efficiencies of the FM design are equal to 74.95 and 80.41, respectively. Considering the Gw -efficiencies, it is the AM design giving a higher G_w - efficiencies than the FM design. In the same manner, the FM design will give a higher value if we consider G -efficiencies. Given a certain criterion, however, robust designs (AM designs) do not differ from the optimal designs (FM designs) in terms of G_w - and G-efficiencies.

An increase in the design efficiency depends on two factors: the total number of design points, N, and the number of design points in each block, n_i. When N increases, the design efficiency tends to increase.

If n_i 's for all blocks are about the same, its design efficiency will be greater than that of a design with different block sizes. For example, in Table 1, the G_w -efficiency of the AM design for N = 8 (both block sizes are 4) equals 75.03 which is greater than that of the AM design for N = 9 (block sizes are 4 and 5, respectively), which equals 73.31.

Table 2 shows a comparison of G-efficiency for each design under the first-order model (FOM), interaction model (INT), and second-order model (SOM). For example, for N = 8, when sample size in the 1st, and 2nd block are both 4, the G-efficiencies of

Table 1: Summary of G_w - and G-efficiencies for "all models" and "full model only" having k = 2 variables and b = 2 blocks

N	n_i	Design	G_w	G	Design Points
8	4 4	AM	75.03	80.40	$(0.35,-1)^1,(-1,1)^1,(-0.78,-0.17)^1,(1,1)^1,$ $(1,-1)^2,(-0.35,1)^2,(-1,-1)^2,(0.78,0.15)^2$
		FM	74.95	80.41	$(0.35,-1)^1,(-1,1)^1,(-0.78,-0.2)^1,(1,1)^1,$ $(1,-1)^2,(-0.35,1)^2,(-1,-1)^2, (0.78,0.15)^2$
9	4 5	AM	73.31	78.91	$(0.7,-0.26)^1,(1,0.9)^1,(-1,1)^1,(-0.33,-1)^1,$ $(-1,-0.08)^2,(1,-1)^2,(0.57,1)^2,(-1,-1)^2,(-0.11,0.8)^2$
		FM	73.14	79.03	$(0.7,-0.26)^1,(1,0.9)^1,(-1,1)^1,(-0.33,-1)^1,$ $(-1,-0.11)^2,(1,-1)^2,(0.55,1)^2,(-1,-1)^2,(-0.11,0.8)^2$
10	5 5	AM	80.00	84.67	$(-1,-0.79)^1,(0.48,1)^1,(1,0)^1,(-1,1)^1,(0.44,-1)^1,$ $(-0.86,0.8)^2,(1,-1)^2,(0.99,1)^2,(-0.87,-1)^2,(-0.21,0)^2$
		FM	79.87	85.17	$(-1,-0.79)^1,(0.44,1)^1,(1,0)^1,(-1,1)^1,(0.4,-1)^1,$ $(-0.86,0.8)^2,(1,-1)^2,(1,1)^2,(-0.89,-1)^2,(-0.21,0)^2$
11	5 6	AM	77.67	81.86	$(1,-0.7)^1,(-0.5,1)^1,(-0.35,-1)^1,(-1,-0.1)^1,(1,0.9)^1,$ $(0.26,0.1)^2,(1,1)^2,(1,-1)^2,(-1,-1)^2,(0.3,0)^2,(-1,1)^2$
		FM	77.66	81.96	$(1,-0.7)^1,(-0.5,1)^1,(-0.35,-1)^1,(-1,0.11)^1,(1,0.88)^1,$ $(0.26,0.1)^2,(1,1)^2,(1,-1)^2,(-1,-1)^2,(0.3,0)^2,(-1,1)^2$
12	6 6	AM	80.66	85.77	$(0.7,-1)^1,(1,0.1)^1,(-1,0.1)^1,(-0.7,-1)^1,(-0.7,1)^1, (0.7,1)^1,$ $(1,-1)^2,(0,0.5)^2,(0,-0.2)^2,(-1,1)^2,(-1,-1)^2,(1,1)^2$
		FM	80.43	85.86	$(0.7,-1)^1,(1,0.12)^1,(-1,0.09)^1,(-0.69,-1)^1,(-0.7,1)^1, (0.7,1)^1,$ $(1,-1)^2,(0,0.5)^2,(0,-0.1)^2,(-1,1)^2,(-1,-1)^2,(1,1)^2$

Table 2: Summary of G-efficiencies for First-order model, Interaction model, and Second-order model from GA designs having $k = 2$ variables and $b = 2$ blocks

k	*b*	*N*	FOM		INT		SOM	
			AM	FM	AM	FM	AM	FM
2	2	8	70.12	69.98	66.65	66.38	80.40	80.41
		9	70.21	70.09	63.92	63.91	78.91	79.03
		10	80.19	79.15	72.41	71.69	84.67	85.17
		11	75.85	75.55	67.07	66.87	81.86	81.96
		12	78.37	77.32	69.31	68.55	85.77	85.86

Table 3: Summary of G_w and G-efficiencies for all models and full model only having $k = 2$ variables and $b = 3$ blocks

N	n_i	Design	G_w	G	Design Points
9	3 3 3	AM	70.64	73.55	$(1,-0.3)^1,(-1,-1)^1,(-0.16,1)^1,(-0.7,0)^2,(1,-1)^2,(1,1)^2,(-1,1)^3,(1,0.3)^3,(-0.16,-1)^3$
		FM	70.54	73.58	$(1,-0.28)^1,(-1,-1)^1,(-0.16,1)^1,(-0.7,0)^2,(1,-1)^2,(1,1)^2,$ $(-1,1)^3,(1,0.3)^3,(-0.16,-1)^3$
10	3 3 4	AM	75.47	78.31	$(1,-1)^1,(-1,-0.41)^1,(0.3,1)^1, (1,1)^2,(0.3,-1)^2,(-1,0.41)^2,$ $(-1,-1)^3,(-0.01,0)^3,(1,0)^3,(-1,1)^3$
		FM	74.79	79.38	$(1,1)^1,(0.43,-1)^1,(-1,0.34)^1,(-1,-0.48)^2,(1,-1)^2,(0.28,1)^2,$ $(1,0.11)^3,(-1,-1)^3,(-1,1)^3,(0.1,-0.2)^3$
11	3 4 4	AM	72.81	75.79	$(1,0.82)^1,(-1,0.55)^1,(0.06,-1)^1,(0.03,0.9)^2,(-1,-0.43)^2,(1,1)^2,(1,-1)^2,$ $(-1,-1)^3,(-1,1)^3,(1,-0.52)^3,(0.14,0.2)^3$
		FM	71.35	77.37	$(1,-1)^1,(0.42,1)^1,(-1,-0.41)^1,(1,1)^2,(0.3,-1)^2,(-1,0.81)^2,(-0.2,-0.4)^2,$ $(0.4,0.2)^3,(1,-0.31)^3,(-0.8,1)^3,(-1,-1)^3$
12	4 4 4	AM	76.34	80.40	$(1,1)^1,(1,-1)^1,(-0.2,0)^1,(-0.92,0)^1,(-1,1)^2,(-0.89,-1)^2,(1,-0.41)^2,$ $(0.3,1)^2,(0.3,-1)^3,(-1,-1)^3,(-0.89,1)^3,(1,0.41)^3$
		FM	76.13	80.41	$(1,1)^1,(1,-1)^1,(-0.2,0)^1,(-0.94,0)^1,(-1,1)^2,(-0.89,-1)^2,(1,-0.44)^2,$ $(0.29,1)^2,(0.22,-1)^3,(-1,-1)^3,(-0.91,1)^3,(1,0.41)^3$
13	4 4 5	AM	77.46	83.34	$(1,1)^1,(-1,1)^1,(-0.17,-1)^1,(1,-0.15)^1,(0.22,1)^2,(-1,-1)^2,(-1,0.18)^2,$ $(1,-1)^2, (0.05,0.25)^3,(1,-1)^3,(1,1)^3,(-1,1)^3,(-1,-1)^3$
		FM	76.97	83.80	$(1,1)^1,(-1,0.98)^1,(-0.12,-1)^1,(1,-0.13)^1,(0.21,1)^2,(-1,-1)^2,(-1,0.05)^2,$ $(1,-1)^2,(0.05,0.25)^3,(1,-1)^3,(1,1)^3,(-1,0.95)^3,(-1,-1)^3$

the AM and FM designs for first-order model (FOM) equal 70.12 and 69.98, respectively. For the interaction model (INT), they are 66.65 and 66.38, respectively. It is concluded that the "usual" G-efficiencies will be greater than those of the FM designs if the robust (AM) designs are used for first-order model and interaction model. That is, the AM designs are more robust to model-misspecification than the FM design for all choices of *N*. Also, the G-efficiencies of the FM designs for the SOM must be greater than those of the AM designs for all choices of N because the goal of the FM designs is to optimize for the full second-order model (SOM) with blocks. However, the G-efficiencies of both AM and FM designs are very close. The same patterns are true for all choices of *k* and *b*.

The results in Tables 3 and 4 are for designs having $k = 2$ variables and $b = 3$ blocks. For example, for $N = 9$ in Table 3, when the sample sizes in 1st, 2nd, and 3rd block are 3, 3, and 3 , respectively, G_w - and G- efficiencies of the AM design equals 70.64 and 73.55, respectively, while the FM design has values 70.54 and 73.58, respectively. If comparing Tables 1 and 3, it can be concluded that both G_w - and G-efficiencies reduce for all choices of *N* as the number of blocks (*b*) increases from 2 to 3 blocks. In Table 4, if we use the AM designs for first-order model and interaction model, the G-efficiencies are greater than those of the FM designs for all choices of *N* similar to the case $b = 2$. We have shown that the AM designs can be very effective in obtaining designs that have higher

Table 4: Summary of G -efficiencies for First-order model, Interaction model, and Second-order model from GA designs having k = 2 variables and b = 3 blocks

k	b	N	FOM		INT		SOM	
			AM	FM	AM	FM	AM	FM
2	3	9	66.03	65.88	61.36	61.29	73.55	73.58
		10	69.72	69.25	65.63	63.77	78.31	79.38
		11	69.98	65.59	64.48	60.84	75.79	77.37
		12	73.06	72.27	66.18	65.98	80.40	80.41
		13	68.86	68.31	67.53	66.51	83.34	83.80

Table 5: Summary of G_w and G-efficiencies for all models and full model only having k = 2 variables and b = 4 blocks

N	n_i	Design	G_w	G	Design Points
10	2, 2, 3, 3	AM	55.18	56.25	$(1,0.17)^1,(-0.66,-1)^1,(0.68,1)^2,(1,-0.22)^2,(-1,1)^3,(-1,-1)^3,(0.2,0)^3,(-1,-0.02)^4,(0.91,1)^4,(0.9,-1)^4$
		FM	54.86	56.27	$(-1,0.13)^1,(0.7,-1)^1,(-1,-0.14)^2,(0.65,1)^2,(1,-1)^3,(1,1)^3,(-0.18,0)^3,(1,-0.02)^4,(-0.91,-1)^4,(-0.92,1)^4$
11	2, 3, 3, 3	AM	59.81	62.60	$(-0.28,1)^1,(1,-0.2)^1,(-1,-1)^2,(-1,1)^2,(0.48,0.2)^2,(0.03,-1)^3,(1,1)^3,(-1,0.19)^3,(0.37,0.7)^4,(-1,-1)^4,(1,-1)^4$
		FM	59.80	62.60	$(-0.28,1)^1,(1,-0.2)^1,(-1,-1)^2,(-1,1)^2,(0.48,0.2)^2,(0.03,-1)^3,(1,1)^3,(-1,0.2)^3,(0.37,0.7)^4,(-1,-1)^4,(1,-1)^4$
12	3, 3, 3, 3	AM	73.78	75.99	$(-0.97,-1)^1,(1,-0.18)^1,(-0.64,1)^1,(1,-1)^2,(-1,-0.88)^2,(-0.03,0.7)^2,(1,0.88)^3,(-1,1)^3,(0.02,-0.7)^3,(-0.99,0.2)^4,(0.98,1)^4,(0.65,-1)^4$
		FM	73.62	76.44	$(-0.96,-1)^1,(1,-0.18)^1,(-0.64,1)^1,(1,-1)^2,(-1,-0.85)^2,(-0.03,0.7)^2,(1,0.83)^3,(-1,1)^3,(0.02,-0.7)^3,(-0.99,0.2)^4,(0.98,1)^4,(0.65,-1)^4$
13	3, 3, 3, 4	AM	74.47	77.85	$(0.96,0.1)^1,(-0.8,-1)^1,(-0.78,1)^1,(1,1)^2,(-1,0.21)^2,(0.3,-1)^2,(-0.83,-0.16)^3,(0.7,1)^3,(0.98,-1)^3,(-1,-1)^4,(0.19,0.4)^4,(1,-0.7)^4,(-1,1)^4$
		FM	74.44	77.88	$(0.96,0.1)^1,(-0.79,-1)^1,(-0.79,1)^1,(1,1)^2,(-1,0.2)^2,(0.3,-1)^2,(-0.83,-0.16)^3,(0.7,1)^3,(0.98,-1)^3,(-1,-1)^4,(0.17,0.4)^4,(1,-0.69)^4,(-1,1)^4$
14	3, 3, 4, 4	AM	75.57	80.02	$(-1,1)^1,(-0.28,-1)^1,(1,0.31)^1,(-1,-1)^2,(-0.25,1)^2,(1,-0.31)^2,(-1,0.9)^3,(-0.23,-0.31)^3,(0.95,-1)^3,(1,1)^3,(0.82,1)^4,(-0.71,0.2)^4,(-1,-1)^4,(0.99,-1)^4$
		FM	75.48	80.09	$(-1,1)^1,(-0.28,-1)^1,(1,0.3)^1,(-1,-1)^2,(-0.25,1)^2,(1,-0.34)^2,(-1,0.9)^3,(-0.23,-0.31)^3,(0.96,-1)^3,(1,1)^3,(0.82,1)^4,(-0.71,0.2)^4,(-1,-1)^4,(0.97,-1)^4$

G-efficiencies than the FM designs when the true model is the first-order model or interaction model.

The results in Tables 5 and 6 are for designs having k = 2 variables and b = 4 blocks. It shows that G_w - and G-efficiencies for b = 4 decrease as the number of blocks increases (see G_w - and G- efficiencies in Tables 1 and 3, respectively) for all choices of N. Similar to the case b = 2 and 3, if the first-order model or interaction model is true, the G-efficiencies of AM designs are greater than those of the FM designs for all choices of N. This means the proposed weighted G-optimal designs in this study are still robust to model misspecification, even if the number of blocks increases.

The designs having k = 3 variables and b = 2, 3, or 4 blocks are presented in Tables 7 to 12. The results have the same pattern as designs with k = 2 variables. Also, the G_w - or G-efficiencies for k = 3 are lower than those for all cases having the same block size and k = 2. Note that in a case of k = 3, b = 3, and N = 13 in Table 9, the G_w -efficiencies of the AM and FM designs are equal to 66.39, although the design points are different. In Table 10, however, if the true model is the first-order or interaction model, the weighted G-optimal design (AM design) has a higher G-efficiency than that of the FM model.

Table 6: Summary of G-efficiencies for First-order model, Interaction model, and Second-order model from GA designs having $k = 2$ variables and $b = 4$ blocks

k	*b*	*N*	FOM		INT		SOM	
			AM	FM	AM	FM	AM	FM
2	4	10	55.10	54.10	46.93	46.90	56.25	56.27
		11	52.86	52.82	51.74	51.72	62.60	62.60
		12	70.24	69.93	65.93	65.10	75.99	76.44
		13	72.05	72.02	68.08	67.95	77.85	77.88
		14	74.25	73.81	69.45	69.13	80.02	80.09

Table 7: Summary of G_w and G-efficiencies for all models and full model only having $k = 3$ variables and $b = 2$ blocks

N	n_i	Design	G_w	G	Design Points
12	6 6	AM	70.27	75.14	$(-0.17,-1,-0.05)^1,(-0.31,0.07,1)^1,(-1,-0.82,-1)^1,(-1,1,0.71)^1,(1,0.95,-1)^1,(1,-0.97,1)^1,(0.8,1,1)^2,(1,0.34,-0.08)^2,(-0.44,1,-1)^2,(-1,0.36,-0.76)^2,(0.92,-0.97,-1)^2,(-1,-1,1)^2$
		FM	69.13	76.08	$(-0.17,-1,0)^1,(-0.3,0.16,1)^1,(-1,-0.83,-1)^1,(-1,1,0.71)^1,(1,0.96,-1)^1,(1,0.91,1)^1,(0.8,1,1)^2,(1,0.45,-0.27)^2,(-0.51,1,-1)^2,(-1,0.52,-0.84)^2,(0.93,-0.95,-1)^2,(-1,-1,1)^2$
13	6 7	AM	71.23	80.44	$(1,0.06,0.15)^1,(-1,1,-0.15)^1,(-0.88,-1,1)^1,(0.98,1,1)^1,(-0.14,0.48,-1)^1,(1,-1,-1)^1,(0.01,-1,0.03)^2,(1,1,-0.9)^2,(1,-1,1)^2,(-1,-1,-1)^2,(-1,0.18,0.94)^2,(-0.47,1,1)^2,(-0.96,0.83,-1)^2$
		FM	71.15	80.65	$(1,0.06,0.16)^1,(-1,1,-0.15)^1,(-0.88,-1,1)^1,(0.97,1,1)^1,(-0.13,0.49,-1)^1,(1,-1,-1)^1,(0.03,-1,0.05)^2,(1,1,-0.9)^2,(1,-1,1)^2,(-1,-0.99,-1)^2,(-1,0.2,0.96)^2,(-0.47,1,1)^2,(-0.94,0.84,-1)^2$
14	7 7	AM	75.15	81.38	$(0.86,1,1)^1,(-0.17,0.09,-1)^1,(1,0.3,0.26)^1,(-1,1,-1)^1,(-1,-1,1)^1,(1,-1,-1)^1,(0.15,-1,-0.11)^1,(1,0.87,-1,1)^2,(-1,-1,-1)^2,(0.11,-0.17,1)^2,(-1,0.13,0.12)^2,(0.28,1,-0.23)^2,(-1,1,1)^2,(1,-1,1)^2$
		FM	73.38	81.66	$(1,1,1)^1,(-0.5,0,-1)^1,(1,-0.2,-0.1)^1,(-1,1,-1)^1,(-1,-0.9,1)^1,(1,-1,-1)^1,(-0.3,-1,0.4)^1,(1,1,-1)^2,(-0.9,-1,-1)^2,(0.1,-0.3,1)^2,(-1,-0.3,-0.1)^2,(-0.1,1,0.4)^2,(-1,1,1)^2,(1,-1,1)^2$
15	7 8	AM	73.80	76.82	$(0.61,0.07,-1)^1,(1,1,1)^1,(-0.85,-1,1)^1,(0.06,-1,-0.89)^1,(-1,1,-1)^1,(1,-0.39,0.16)^1,(-1,0.32,0.55)^1,(1,1,-1)^2,(0.04,-0.21,1)^2,(-1,-0.5,-1)^2,(1,-1,1)^2,(-1,1,1)^2,(1,-1,-0.88)^2,(0.1,1,-0.03)^2,(-1,-1,-0.21)^2$
		FM	72.28	78.41	$(0.35,-0.11,-1)^1,(1,1,1)^1,(-0.85,-1,1)^1,(0,-1,-1)^1,(-1,1,-1)^1,(1,-0.24,0.15)^1,(-1,0.2,0.4)^1,(1,1,-1)^2,(0.03,-0.1,1)^2,(-1,-0.6,-1)^2,(1,-1,1)^2,(-1,1,1)^2,(1,-1,-0.9)^2,(0.03,1,-0.15)^2,(-1,-1,-0.1)^2$
16	8 8	AM	78.05	81.65	$(1,0.35,-0.35)^1,(-0.98,-1,1)^1,(0,-1,-0.06)^1,(1,-1,-1)^1,(-1,0.43,-0.47)^1,(-0.94,1,-1)^1,(1,1,1)^1,(-0.03,0.12,1)^1,(0.98,1,-1)^2,(-1,1,1)^2,(-1,-0.35,0.35)^2,(1,-0.43,0.47)^2,(0.03,-0.12,-1)^2,(0,1,0.06)^2,(0.94,-1,1)^2,(-1,-1,-1)^2$
		FM	75.94	81.78	$(1,0.31,-0.34)^1,(-0.99,-1,1)^1,(-0.2,-1,-0.23)^1,(1,-1,-1)^1,(-1,0.45,-0.46)^1,(-0.95,1,-1)^1,(1,1,1)^1,(0.03,0.18,1)^1,(0.99,1,-1)^2,(-1,1,1)^2,(-1,-0.32,0.3)^2,(1,-0.39,0.44)^2,(-0.05,-0.27,-1)^2,(0.25,1,0.3)^2,(0.96,-1,1)^2,(-1,-1,-1)^2$

Table 8: Summary of G-efficiencies for First-order model, Interaction model, and Second-order model from GA designs having $k = 3$ variables and $b = 3$ blocks

k	*b*	*N*	FOM		INT		SOM	
			AM	FM	AM	FM	AM	FM
3	2	12	73.36	71.88	61.84	61.61	75.14	76.08
		13	69.67	69.24	62.33	62.04	80.44	80.65
		14	65.65	64.75	61.50	61.49	81.38	81.66
		15	75.90	72.73	60.92	60.32	76.82	78.41
		16	77.24	76.66	63.91	63.88	81.65	81.78

Table 9: Summary of G_w and G-efficiencies for all models and full model only having k = 3 variables and b = 3 blocks

N	n_i	Design	G_w	G	Design Points
13	4 4 5	AM	66.39	72.49	$(1,1,0.31)^1,(1,-1,1)^1,(-1,0.54,0.7)^1,(-0.06,-0.27,-1)^1,$ $(1,-0.34,0.28,1)^2,(-0.46,0.98,1)^2,(0.81,1,-1)^2,(-1,-1,-1)^2,$ $(-1,1,-0.84)^3,(1,0.51,1)^3,(0.01,-0.99,0.1)^3,(1,-1,-1)^3,(-1,-1,1)^3$
		FM	66.39	72.58	$(1,1,0.31)^1,(1,-1,1)^1,(-1,0.54,0.69)^1,(-0.07,-0.25,-1)^1,$ $(1,-0.34,0.28)^2,(-0.46,0.98,1)^2,(0.81,1,-1)^2,(-1,-1,-1)^2,$ $(-1,1,-0.83)^3,(1,0.51,1)^3,(0.01,-0.96,0.1)^3,(1,-1,-1)^3,(-1,-1,1)^3$
14	4 5 5	AM	68.50	71.35	$(1,0.07,0.35)^1,(-1,0.97,1)^1,(-0.44,1,-1)^1,(-0.27,-1,-0.16)^1,$ $(0.97,-0.97,-1)^2, (1,1,1)^2,(-0.23,0.99,0.1)^2,(-1,0.27,-1)^2,(-1,-1,1)^2,$ $(1,-1,1)^3,(-1,-1,-1)^3,(-1,1,0.09)^3,(-0.2,0.25,1)^3,(1,0.91,-1)^3$
		FM	67.33	73.75	$(1,0.09,0.37)^1,(-1,1,1)^1,(-0.45,1,-1)^1,(-0.32,-1,0.12)^1,$ $(0.98,-0.96,-1)^2,(1,1,1)^2,(-0.23,0.99,0.1)^2,(-1,0.32,1)^2,(-1,-1,1)^2,$ $(1,-1,1)^3,(-1,-1,-1)^3,(-1,0.88,0.11)^3,(-0.22,0.21,1)^3,(1,0.92,-1)^3$
15	5 5 5	AM	71.06	77.99	$(-0.98,-1,-1)^1,(1,0.45,-0.63)^1,(-1,1,-0.04)^1,(1,-1,1)^1,(-0.29,0.19,1)^1,$ $(0.97,-1,-1)^2,(-0.32,1,-1)^2,(-1,-1,1)^2,(-1,0.34,-0.43)^2,(1,1,1)^2,$ $(1,-0.35,0.73)^3,(-1,0.35,-1)^3,(-1,1,1)^3,(-0.12,-1,-0.06)^3,(1,1,-1)^3$
		FM	71.04	78.12	$(-0.98,-1,-1)^1,(1,0.62,-0.67)^1,(-1,1,-0.01)^1,(1,-1,1)^1,(-0.23,0.22,1)^1,$ $(0.98,-1,-1)^2,(-0.31,1,-1)^2,(-1,-1,1)^2,(-1,0.4,-0.27)^2,(1,1,1)^2,$ $(1,-0.12,0.79)^3,(-1,0.41,-1)^3,(-1,1,1)^3,(-0.21,-1,-0.03)^3,(1,1,-1)^3$
16	5 5 6	AM	71.62	73.42	$(1,-1,-1)^1,(1,0.74,0.72)^1,(-0.37,1,-0.29)^1,(-0.6,-1,1)^1,(-1,0.01,-1)^1,$ $(1,1,-1)^2,(-1,-1,-1)^2,(-1,1,1)^2,(1,-1,1)^2,(-0.1,-0.1,0)^2,$ $(0.12,-0.6,-1)^3,(-1,-0.42,1)^3,(-1,1,-1)^3, (1,-0.39,-0.3)^3,(-1,-1,0.05)^3, (0.75,1,1)^3$
		FM	70.32	76.07	$(1,-1,-1)^1,(1,0.6,0.4)^1,(-0.11,1,-0.11)^1,(-0.7,-1,1)^1,(-1,-0.1,-1)^1,$ $(1,1,-1)^2,(-1,-1,-0.9)^2,(-1,1,1)^2,(1,-1,1)^2,(-0.1,-0.1,0)^2,$ $(0.07,-0.6,-1)^3,(-1,-0.5,1)^3,(-1,1,-1)^3, (1,-0.51,-0.51)^3,(-1,-1,0), (0.9,1,1)^3$
17	5 6 6	AM	71.79	76.25	$(1,-0.33,1)^1,(0.83,1,-1)^1,(-1,-0.46,-0.73)^1,(0.05,-1,-0.01)^1,$ $(-0.94,1,1)^1,(-0.91,1,-1)^2,(-0.16,0.4,0)^2,(1,-0.17,-0.32)^2,(1,-1,-1)^2,$ $(0.81,1,1)^2,(-1,-1,1)^2, (0.35,-0.16,-1)^3,(-1,0.85,0.13)^3,(1,-1,1)^3,$ $(1,1,-0.15)^3,(-0.44,-0.11,1)^3,(-1,-1,-1)^3$
		FM	70.99	76.30	$(1,-0.39,1)^1,(0.84,1,-1)^1,(-1,-0.32,-0.76)^1,(-0.05,-1,-0.23)^1,$ $(-0.91,1,1)^1,(-0.9,1,-1)^2,(-0.16,0.55,0)^2,(1,-0.28,-0.52)^2,$ $(1,-1,-1)^2,(0.85,1,1)^2,(-1,-1,1)^2,(0.16,-0.17,-1)^3,(-1,0.91,0.25)^3,$ $(1,-1,1)^3,(1,1,-0.21)^3,(-0.5,-0.26,1)^3,(-1,-1,-1)^3$

Table 10: Summary of G-efficiencies for First-order model, Interaction model, and Second-order model from GA designs having k = 3 variables and b = 3 blocks

k	b	N	FOM		INT		SOM	
			AM	FM	AM	FM	AM	FM
3	3	13	65.02	64.88	58.72	58.52	72.49	72.58
		14	66.97	63.07	61.45	58.81	71.35	73.75
		15	75.23	74.28	62.71	61.05	77.99	78.12
		16	74.91	71.52	60.62	60.13	73.42	76.07
		17	74.33	73.79	61.28	60.57	76.25	76.30

Table 11: Summary of G_w and G-efficiencies for all models and full model only having k = 3 variables and b = 4 blocks

N	n_i	Design	G_w	G	Design Points
14	3344	AM	58.31	62.02	$(-1,-1,1)^1,(0.78,1,0.32)^1,(0.25,-0.3,-1)^1,(-0.34,-0.2,0.99)^2,(-0.99,0.85,-0.65)^2,(1,-1,-1)^2,(1,-0.08,-0.14)^3,(-1,1,-1)^3,(-0.16,-1,0.04)^3,(1,1,1)^3,(0.9,1,-1)^4,(-1,-0.93,-0.89)^4,(1,-1,1)^4,(-1,0.95,0.94)^4$
		FM	58.28	62.19	$(-1,-1,1)^1,(0.77,1,0.3)^1,(0.27,-0.28,-1)^1,(-0.34,-0.2,0.99)^2,(-0.99,0.85,-0.65)^2,(1,-1,-1)^2,(1,-0.11,-0.13)^3,(-1,1,-1)^3,(-0.16,-1,0.03)^3,(1,1,1)^3,(0.9,1,-1)^4,(-1,-0.93,-0.89)^4,(1,-1,1)^4,(-1,0.95,0.94)^4$
15	3444	AM	60.22	62.83	$(-1,1,-0.05)^1,(-0.17,-1,1)^1,(1,0.07,-0.79)^1,(-0.9,0.87,1)^2,(0.99,-0.93,0.4)^2,(-1,-1,-1)^2,(0.66,1,-1)^2,(-1,0.94,-1)^3,(1,-1,-0.56)^3,(-0.38,-0.07,0)^3,(1,1,1)^3,(0.49,1,0.1)^4,(0.15,-1,-1)^4,(-1,-1,1)^4,(0.96,-0.29,1)^4$
		FM	60.21	62.85	$(-1,1,-0.05)^1,(-0.17,-1,1)^1,(1,0.07,-0.79)^1,(-0.89,0.88,1)^2,(0.99,-0.93,0.4)^2,(-1,-1,-1)^2,(0.66,1,-1)^2,(-1,0.93,-1)^3,(1,-1,-0.56)^3,(-0.38,-0.07,0)^3,(1,1,1)^3,(0.49,1,0.1)^4,(0.15,-1,-1)^4,(-1,-1,1)^4,(0.96,-0.29,1)^4$
16	4444	AM	69.03	71.92	$(-1,1,0.31)^1,(-0.83,-0.94,-1)^1, (1,1,-1)^1,(0.53,-0.15,0.95)^1,(-0.86,1,1)^2,(0.7,0.63,0.35)^2,(-0.95,0.78,-0.95)^2,(1,-1,-1)^2,(1,-1,1)^3,(-1,-0.85,-0.45)^3,(-0.23,1,1)^3, (0.97,0.22,-0.88)^3,(0.5,-1,0.07)^4,(-1,0.19,1)^4,(-0.29,0.84,-1)^4,(1,1,1)^4$
		FM	68.88	72.31	$(-1.,1,0.31)^1,(-0.84,-0.96,-1)^1,(1,1,-1)^1, (0.53,-0.15,0.95)^1,(-0.87,-1,1)^2,(0.7,0.63,0.35)^2,(-0.95,0.78,-0.95)^2,(1,-1,-1)^2,(1,-1,1)^3,(-1,-0.86,-0.48)^3,(-0.24,1,1)^3,(0.97,0.22,-0.88)^3,(0.54,-1,0.09)^4,(-1,0.21,1)^4,(-0.27,0.88,-1)^4,(1,1,1)^4$
17	4445	AM	69.59	72.81	$(1,0.94,1)^1,(-0.1,1,-1)^1,(1,-1,-0.89)^1,(-1,-0.33,0.32)^1,(-1,1,-0.55)^2, (-1,-1,1)^2,(0.71,0.24,1)^2,(0.64,-0.97,-1)^2,(-1,0.14,-1)^3,(1,1,-0.07)^3,(-1,0.61,1)^3,(0.06,-1,0.84)^3,(-1,-1,-1)^4,(-0.59,1,1)^4,(1,0.68,-1)^4,(1,-1,1)^4, (0.3,-0.36,0)^4$
		FM	68.34	73.18	$(1,0.9,1)^1,(-0.05,1,-1)^1,(1,-1,-0.95)^1,(-1,-0.24,0.33)^1,(-1,1,-0.56)^2,(-1,-1,1)^2,(0.68, 0.36,1)^2,(0.65,-0.98,-1)^2,(-1,0.19,-1)^3,(1, 1,0.1)^3,(-1,0.56,1)^3,(0.06,-1,0.9)^3,(-1,-1,-1)^4,(-0.59,1,1)^4,(1,0.76,-1)^4,(1,-1,1)^4,(0.3,-0.36,0)^4$
18	4455	AM	69.73	71.80	$(0.18,1,-1)^1,(-1,0.61,1)^1,(1,-0.54,0.4)^1,(-1,-1,-0.94)^1,(-0.51,-1,-0.68)^2,(1,-0.55,1)^2,(-1,1,0.21)^2,(1,0.37,-1)^2,(1,-1,-1)^3,(1,1,1)^3,(-0.27,0.03,0)^3,(-1,1,-1)^3,(-1,-1,1)^3,(1,-1,0.48)^4,(-0.34,-1,1)^4,(-0.4,1,1)^4,(1,1,-0.54)^4,(-0.99,-0.18,-1)^4$
		FM	67.40	72.42	$(-0.11,1,-1)^1,(-1,0.74,1)^1,(1,-0.65,0.11)^1, (-1,-1,-0.96)^1,(-0.47,-1,-0.73)^2,(1,-0.55,1)^2,(-1,1,0.03)^2,(1,0.36,-1)^2,(1,-1,-1)^3,(1,1,1)^3,(-0.27,0.03,0)^3,(-1,1,-1)^3,(-1,-1,1)^3,(1,-1,0.57)^4,(-0.31,-1,1)^4,(-0.32,1,1)^4, (1,1,-0.57)^4,(-0.85,-0.33,-1)^4$

Table 12: Summary of G-efficiencies for First-order model, Interaction model, and Second-order model from GA designs having k = 3 variables and b = 4 blocks

k	b	N	FOM		INT		SOM	
			AM	FM	AM	FM	AM	FM
3	4	14	67.76	67.52	52.19	52.09	62.02	62.19
		15	67.05	66.98	51.03	50.99	62.83	62.85
		16	68.72	68.04	61.26	61.19	71.92	72.31
		17	67.15	65.88	61.26	61.16	72.81	73.18
		18	71.89	67.78	60.94	58.51	71.80	72.42

Based on the results in this paper, it can be confirmed that the robust designs optimizing weighted G-optimality of all models are considered more robust than the FM designs, which are only the optimal design for the second-order model. Thus, when a response surface experiment is to be run, the AM designs guarantee all potential weak heredity models can be fit while little is lost in terms of G-efficiency even when the second-order model is the correct model.

5 Conclusions

The results in this paper support the idea that optimal designs (for only the second-order model) may be more inefficient than previously thought, meaning one might doubt them. As we cannot ignore the uncertainty of the possible reduced models prior to data collection, researchers should consider using experimental designs that are more robust across the set of potential models. The proposed robust design (G_w -optimal designs) could be another suitable option for researchers. Even if a true model is a second-order model, it is not necessary to use a G-optimal (FM) design because the corresponding G-efficiency of the robust (AM) design for the second-order model is very close to that of the FM design. That is, very little G-efficiency is lost using the AM design when the second-order model is true. Current research conducted by the authors indicates similar results also hold using a weighted D-optimality criterion. Future research will be conducted to extend the scope to include model-robust four-factor (k = 4) response surface designs with blocks.

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