Research Article

New Designing Mixed Double Moving Average-Cumulative Sum Control Chart for Detecting Mean Shifts with Symmetrically and Asymmetrically Distributed Observations

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Abstract
Herein, we present a new control chart called the mixed double moving average-cumulative sum control chart (DMA-CUSUM: MDC) used for detecting shifts in the process mean when symmetrically and asymmetrically distributed. The performance of the MDC chart is compared with shewhart, cumulative sum (CUSUM), double moving average (DMA) and mixed cumulative sum-double moving average (CUSUM-DMA: MCD) control charts by using average run length (ARL) and median run length (MRL) with the monte carlo simulation (MC). The research results show that the proposed (MDC) control chart was more efficient than the Shewhart, CUSUM, DMA and MCD charts for all distributions tested. We apply the MDC chart to real sets of data: I) the tensile data of single carbon fiber and II) the survival times of guinea pigs infected with virulent bacilli.

Keywords: Double moving average control chart, Cumulative sum control chart, Double moving average-cumulative sum control chart, Average run length, Median run length

1 Introduction

Control charts are an important tool of statistical process control (SPC), for controlling, monitoring, and improving processes. Applying control charts to various tasks, such as manufacturing processes, health care, environmental science, finance, etc. Shewhart [1] proposed the original control chart, which is called the Shewhart control chart. Shewhart chart is sensitive to detect small-to-moderate changes in the process, but it is good for detecting large changes in the process. Later, Page [2] presented the CUSUM control chart for detecting small changes in the process. Next, Khoo [3] presented the moving average (MA) control chart to detect small changes in the process. Khoo *et al.* [4] presented the DMA control chart for the monitoring of normality distributed quality characteristics. Zaman *et al.* [5] proposed the mixed cumulative sum-exponentially weighted moving average (MCE) control chart. Aslam *et al.* [6] proposed the double moving average-exponentially weighted moving average (DMA-EWMA) control chart for exponentially distributed quality. Ajadi and Riaz [7] proposed the mixed multivariate EWMA-CUSUM control charts for improved process monitoring. Abbas *et al.* [8] proposed the mixed EWMA dual-CUSUM control chart and its applications. Thitisoowaranon *et al.* [9] presented the mixed cumulative sum - Tukey's (CUSUM-TCC) chart for detecting process dispersion. Alves *et al.* [10] proposed the mixed CUSUM-EWMA (MCE) control chart as a new alternative for monitoring a manufacturing process. Hussain *et al.* [11] proposed a class of mixed EWMA-CUSUM median control charts for process monitoring. Abid *et al.* [12] proposed a mixed HWMA-CUSUM mean chart with an application to the manufacturing process. Phantu and Sukparungsee [13] proposed a mixed double exponentially weighted moving average - Tukey's (MDEWMA-TCC) control chart for monitoring of parameter change. Saengsura *et al.* [14] proposed the mixed moving average-cumulative sum (MA-CUSUM: MMC) control chart for monitoring parameter change. Later, Sukparungsee *et al.* [15] proposed the mixed Tukey - double moving average (Tukey-DMA) control chart for monitoring the process mean.

Therefore, we presented the new mixed control chart called the mixed Double Moving Average-Cumulative Sum (DMA-CUSUM: MDC) control chart used for detecting shifts in the process mean when symmetrically and asymmetrically distributed. Performance comparison of the proposed chart versus Shewhart, CUSUM, DMA and CUSUM-DMA control charts, if any control chart has the lowest ARL and MRL, it means that a chart has the best efficiency. Moreover, we apply it to real sets of data: the tensile data of single carbon fiber and the survival times of guinea pigs infected with virulent bacilli.

2 The Designing of Control Charts

The researchers apply the parametric control charts to this research, which were CUSUM and DMA control charts. We proposed a new control chart combining CUSUM and DMA control charts (CUSUM-DMA: MCD, DMA-CUSUM: MDC) as follows.

2.1 *Cumulative sum control chart (CUSUM)*

The CUSUM control chart was presented by Page, it is an effective control chart for detecting small and moderate shifts in processes. This control chart has two statistics C_i^+ and C_i^-, which are defined in Equation (1) below,

$$C_i^+ = \max(0, C_{i-1}^+ + Y_i - \mu_0 - d)$$
$$C_i^- = \min(0, C_{i-1}^- + Y_i - \mu_0 + d) \quad (1)$$

where C_i^+ and C_i^- are the statistics values of the CUSUM control chart and we set $C_i^+ = C_i^- = 0$, Y_i is the observation at the time i, μ_0 is the mean of process, and d is the reference value. The control limits of the CUSUM chart as shown in Equation (2) below,

$$UCL = H_1$$
$$LCL = -H_1 \quad (2)$$

where H_1 is the coefficient of the control limits of the CUSUM control chart.

2.2 *Double moving average control chart (DMA)*

The DMA control chart was presented by Khoo *et al.*, this sensitive control chart for monitoring small and moderate changes in processes. The values of DMA statistics are the collected double moving average of the MA statistic. The DMA statistic of span w at the time i as shown in Equation (3) below,

$$DMA_i = \begin{cases} \dfrac{MA_i + MA_{i-1} + MA_{i-2} + \dots}{i} & ; i \le w \\ \dfrac{MA_i + MA_{i-1} + \dots + MA_{i-w+1}}{w} & ; w < i < 2w-1 \\ \dfrac{MA_i + MA_{i-1} + \dots + MA_{i-w+1}}{w} & ; i \ge 2w-1 \end{cases} \quad (3)$$

where MA_i is a statistic of the MA control chart. Assume Y_1, Y_2,... where $Y_i \sim N(\mu, \sigma^2)$ for $i = 1, 2,...$ is obtained from the process. The MA statistic of span w at times i as shown in Montgomery [16]

$$MA_i = \begin{cases} \dfrac{Y_i + Y_{i-1} + Y_{i-2} + \dots}{i} & ; i < w \\ \dfrac{Y_i + Y_{i-1} + \dots + Y_{i-w+1}}{w} & ; i \ge w \end{cases}. \quad (4)$$

The mean and variance of the DMA control chart are indicated in Equations (5) and (6) below,

$$E(DMA_i) = \mu_0 \quad (5)$$

and

$$Var(DMA_i) = \begin{cases} \dfrac{\sigma^2}{i} & ; i \le w \\ \dfrac{\sigma^2}{w} & ; w < i < 2w-1. \\ \dfrac{\sigma^2}{w} & ; i \ge 2w-1 \end{cases} \quad (6)$$

The control limits of the DMA chart as shown in Equation (7) below,

$$ULC/LCL = \begin{cases} \mu_0 \pm H_2 \dfrac{\sigma}{\sqrt{i}} & ; i \le w \\ \mu_0 \pm H_2 \dfrac{\sigma}{\sqrt{w}} & ; w < i < 2w-1 \\ \mu_0 \pm H_2 \dfrac{\sigma}{\sqrt{w}} & ; i \ge 2w-1 \end{cases} \quad (7)$$

where μ_0 and σ are the mean and standard deviation of the process, respectively. H_2 is the coefficient of the control limits of the DMA control chart.

2.3 *Mixed cumulative sum-double moving average control chart (CUSUM-DMA: MCD)*

The CUSUM control chart combined with the DMA control chart is called CUSUM-DMA or MCD control chart. The statistic is that of the CUSUM control chart as indicated in Equation (1), the control limits values of the MCD control chart is the data expectation value, which is the same value as of the DMA control chart. The variance is applied with the combination of CUSUM and DMA control charts. The MCD statistic of span w at times i as shown in Equations (8) and (9) below,

$$MCD_i^+ = \begin{cases} \dfrac{MCM_i^+ + MCM_{i-1}^+ + MCM_{i-2}^+ + \ldots}{i} & ; i \le w \\ \dfrac{MCM_i^+ + MCM_{i-1}^+ + \ldots + MCM_{i-w+1}^+}{w} & ; w < i < 2w-1 \\ \dfrac{MCM_i^+ + MCM_{i-1}^+ + \ldots + MCM_{i-w+1}^+}{w} & ; i \ge 2w-1 \end{cases} \tag{8}$$

and

$$MCD_i^- = \begin{cases} \dfrac{MCM_i^- + MCM_{i-1}^- + MCM_{i-2}^- + \ldots}{i} & ; i \le w \\ \dfrac{MCM_i^- + MCM_{i-1}^- + \ldots + MCM_{i-w+1}^-}{w} & ; w < i < 2w-1. \\ \dfrac{MCM_i^- + MCM_{i-1}^- + \ldots + MCM_{i-w+1}^-}{w} & ; i \ge 2w-1 \end{cases} \tag{9}$$

The control limits of the MCD chart as shown in Equations (10) and (11) below,

$$UCL = \begin{cases} \mu_0 + H_3 \dfrac{\sigma}{\sqrt{mi^2}} \sqrt{\sum_{j=1}^{i} \dfrac{1}{j}} & ; i \le w \\ \mu_0 + H_3 \dfrac{\sigma}{\sqrt{mw^2}} \sqrt{\sum_{j=1-w+1}^{w-1} \dfrac{1}{j} + (j-w+1)(\dfrac{1}{w})} & ; w < i < 2w-1 \\ \mu_0 + H_3 \dfrac{\sigma}{\sqrt{mw^2}} & ; i \ge 2w-1 \end{cases} \tag{10}$$

and

$$LCL = \begin{cases} \mu_0 - H_3 \dfrac{\sigma}{\sqrt{mi^2}} \sqrt{\sum_{j=1}^{i} \dfrac{1}{j}} & ; i \le w \\ \mu_0 - H_3 \dfrac{\sigma}{\sqrt{mw^2}} \sqrt{\sum_{j=1-w+1}^{w-1} \dfrac{1}{j} + (j-w+1)(\dfrac{1}{w})} & ; w < i < 2w-1 \\ \mu_0 - H_3 \dfrac{\sigma}{\sqrt{mw^2}} & ; i \ge 2w-1 \end{cases} \tag{11}$$

where μ_0 is the mean of the process, σ is the standard deviation of the process and H_3 is the coefficient of the control limits of the MCD control chart.

2.4 *Mixed double moving average-cumulative sum control chart (DMA-CUSUM: MDC)*

The DMA control chart is combined with the CUSUM control chart and is called DMA-CUSUM or MDC control chart. The statistic is that of the DMA control chart as shown in Equation (3), the control limit values of MDC control chart is the data expectation value, which is the same value with of CUSUM control chart. The variance is applied with the combination of DMA and CUSUM control charts. The MCD statistic as shown in Equation (12) below,

$$\begin{aligned} MDC_i^+ &= \max(0, MDC_{i-1}^+ + DMA_i - \mu_0 - d) \\ MDC_i^- &= \min(0, MDC_{i-1}^- + DMA_i - \mu_0 + d) \end{aligned} \tag{12}$$

The control limits of the MCD control chart as shown in Equations (13) and (14) below,

$$UCL = \begin{cases} H_4 \dfrac{\sigma}{\sqrt{mi^2}} \sqrt{\sum_{j=1}^{i} \dfrac{1}{j}} & ; i \le w \\ H_4 \dfrac{\sigma}{\sqrt{mw^2}} \sqrt{\sum_{j=1-w+1}^{w-1} \dfrac{1}{j} + (j-w+1)(\dfrac{1}{w})} & ; w < i < 2w-1 \\ H_4 \dfrac{\sigma}{\sqrt{mw^2}} & ; i \ge 2w-1 \end{cases} \tag{13}$$

and

$$LCL = \begin{cases} -H_4 \dfrac{\sigma}{\sqrt{mi^2}} \sqrt{\sum_{j=1}^{i} \dfrac{1}{j}} & ; i \le w \\ -H_4 \dfrac{\sigma}{\sqrt{mw^2}} \sqrt{\sum_{j=1-w+1}^{w-1} \dfrac{1}{j} + (j-w+1)(\dfrac{1}{w})} & ; w < i < 2w-1 \\ -H_4 \dfrac{\sigma}{\sqrt{mw^2}} & ; i \ge 2w-1 \end{cases} \tag{14}$$

where σ is a standard deviation of the process and H_4 is the coefficient of the control limits of the MDC control chart.

3 Performance Comparison Methods

The most popular method to compare the performance of control charts is Average run length (ARL) [17]. This is an average number of observations required

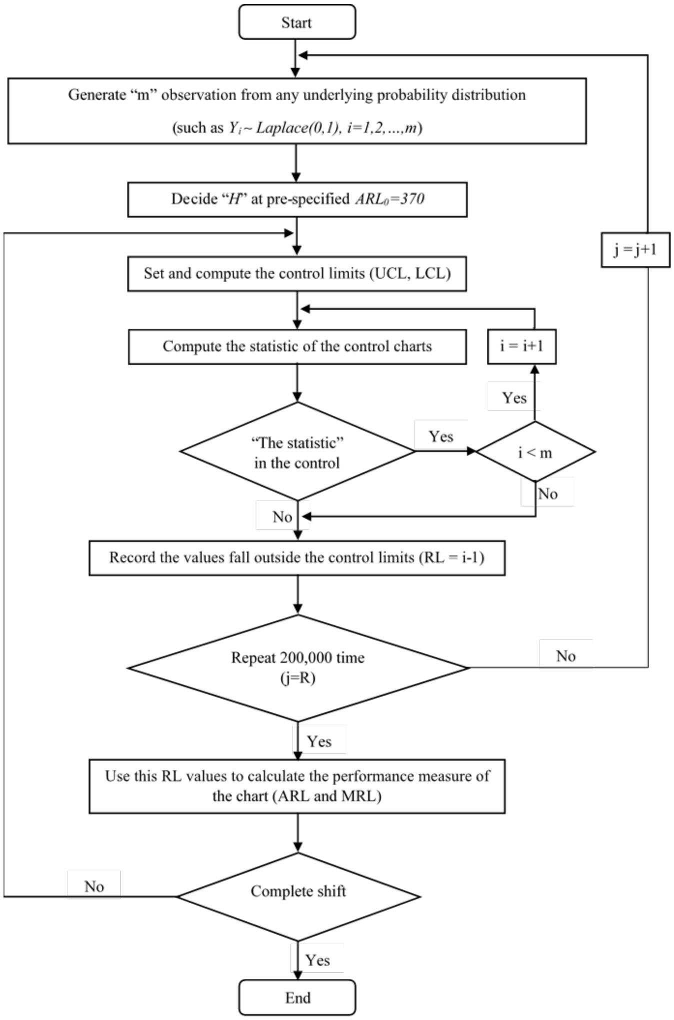

Figure 1: Flow chart of finding ARL and MRL with MC for control charts.

to be monitored before an out-of-control process is detected for the first time. We can consider the ARL in 2 cases: ARL_0 when the process is in-control and ARL_1 when the process is out-of-control. Also, the median run length (MRL) [18] is used as the criteria for measuring the efficiency of the non-normality cases.

In this research, the ARL and MRL are used as the criteria for measuring the efficiency of the control charts, which are evaluated by using Monte Carlo simulation (MC), as shown in Equations (15) and (16) below,

$$ARL = \frac{\sum_{j=1}^{R} RL_j}{R} \tag{15}$$

and

$$MRL = Median(RL) \tag{16}$$

where RL_j is the numbers of samples before the process is out of control for the first time in simulating the data of j, R is the number of repetitions in the simulation, and the values are set as follows: I) set $m = 5{,}000$; where m is the sample size of each round of the experiment, II) set $R = 200{,}000$; where R is the number of the experiment repetition, and III) set $ARL_0 = 370$; when the process is in-control, as shown on Figure 1.

4 Research Results

We compared the performance of the MDC control chart versus Shewhart, CUSUM, DMA, and MCD control charts. The Normal (0,1), Laplace (0,1), Exponential (1), and Gamma (4,1) distributions are patterns of the in-control processes. In addition, we compared their performances while detecting change $\delta \in [-4, 4]$. The most efficient control charts have the lowest ARL and MRL values.

The first process with normal distribution when $\mu = 0$, $\sigma^2 = 1$ as shown in Table 1, Figures 2 and 3. The MDC control chart provided $H_4 = 14.486$ and the lowest ARL_1 and MRL values for $\delta = \pm 0.05, \pm 0.10, \pm 0.25, \pm 0.50, \pm 0.75, \pm 1.00$, and ± 1.50. The CUSUM control chart provided $H_1 = 4.100$ and the lowest ARL_1 and MRL values for $\delta = \pm 2.00$ and ± 3.00. Also, the DMA control chart provided $H_2 = 5.001$ and the lowest ARL_1 and MRL values for $\delta = \pm 4.00$.

The second process with Laplace distribution when $\alpha = 0$, $\beta = 1$, as shown in Table 2, Figures 2 and 3. The MDC control chart provided $H_4 = 20.982$ and the lowest ARL_1 and MRL values for $\delta = \pm 0.05$, $\pm 0.10, \pm 0.25, \pm 0.50, \pm 0.75, \pm 1.00, \pm 1.50$ and ± 2.00. Also, the CUSUM control chart provided $H_1 = 7.845$ and the lowest ARL_1 and MRL values for $\delta = \pm 3.00$ and ± 4.00.

The third process with exponential distribution when $\lambda = 0$, as shown in Table 3, Figures 2 and 3. The MDC control chart provided and the lowest ARL_1 and MRL values for $\delta = \pm 0.05, \pm 0.10, \pm 0.25, \pm 0.50, \pm 0.75$ and ± 1.00. And the CUSUM control chart provided $H_1 = 6.128$ and the lowest ARL_1 and MRL values for $\delta = \pm 1.50, \pm 2.00$ ± 3.00 and ± 4.00.

The fourth process with gamma distribution when $\alpha = 4$, $\beta = 1$, as shown in Table 4, Figures 2 and 3. The MDC control chart provided $H_4 = 20.151$ and the lowest ARL_1 and MRL values than the others at all change levels.

Table 1: ARL and MRL performance of the MDC vs Shewhart, CUSUM, DMA, and MCD control chart for normal distribution

δ	Shewhart		CUSUM		DMA		MCD		MDC	
	$H = 3.00$		$H_1 = 4.100$		$H_2 = 5.001$		$H_3 = 14.245$		$H_4 = 14.486$	
	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL
–4.00	1.19	1.00	1.02	1.00	0.96	1.00	3.48	3.00	1.35	1.00
–3.00	2.00	1.00	1.29	1.00	1.67	2.00	4.37	4.00	1.86	1.00
–2.00	6.28	4.00	2.42	2.00	3.18	3.00	5.45	5.00	**2.76**	**2.00**
–1.50	14.97	11.00	3.85	3.00	4.17	5.00	6.61	6.00	**3.76**	**3.00**
–1.00	43.93	31.00	7.59	6.00	9.45	7.00	9.88	9.00	**6.21**	**5.00**
–0.75	81.31	56.00	12.67	10.00	17.49	13.00	14.58	12.00	**9.70**	**7.00**
–0.50	155.53	108.00	26.75	20.00	42.20	30.00	27.78	21.00	**20.11**	**13.00**
–0.25	28.07	196.00	80.91	57.00	140.57	98.00	79.02	57.00	**65.17**	**42.00**
–0.10	352.43	246.00	192.36	134.00	297.99	206.00	188.45	133.00	**172.36**	**111.00**
–0.05	366.14	255.00	265.03	184.00	349.17	241.00	262.46	184.00	**259.99**	**163.00**
0.00	370.62	258.00	370.02	257.00	370.11	256.00	370.06	259.00	370.08	243.00
0.05	366.29	255.00	264.89	184.00	348.83	242.00	**262.24**	184.00	279.64	**162.00**
0.10	352.89	246.00	192.69	134.00	297.68	206.00	188.41	133.00	**171.80**	**111.00**
0.25	281.55	195.00	80.80	57.00	140.88	98.00	79.09	57.00	**65.19**	**42.00**
0.50	155.24	107.00	26.75	20.00	42.16	30.00	27.72	21.00	**20.03**	**13.00**
0.75	81.34	57.00	12.65	10.00	17.39	13.00	14.55	12.00	**9.67**	**7.00**
1.00	43.88	31.00	7.59	6.00	9.43	7.00	9.87	9.00	**6.21**	**5.00**
1.50	14.95	10.00	3.85	3.00	4.72	5.00	6.61	6.00	**3.76**	**3.00**
2.00	6.31	5.00	2.41	2.00	3.19	3.00	5.45	5.00	**2.76**	**2.00**
3.00	2.00	2.00	1.29	1.00	**1.68**	2.00	4.37	4.00	1.86	2.00
4.00	1.19	1.00	1.02	1.00	**0.96**	1.00	3.48	3.00	1.35	1.00

The minimal values of ARL and MRL are bold.

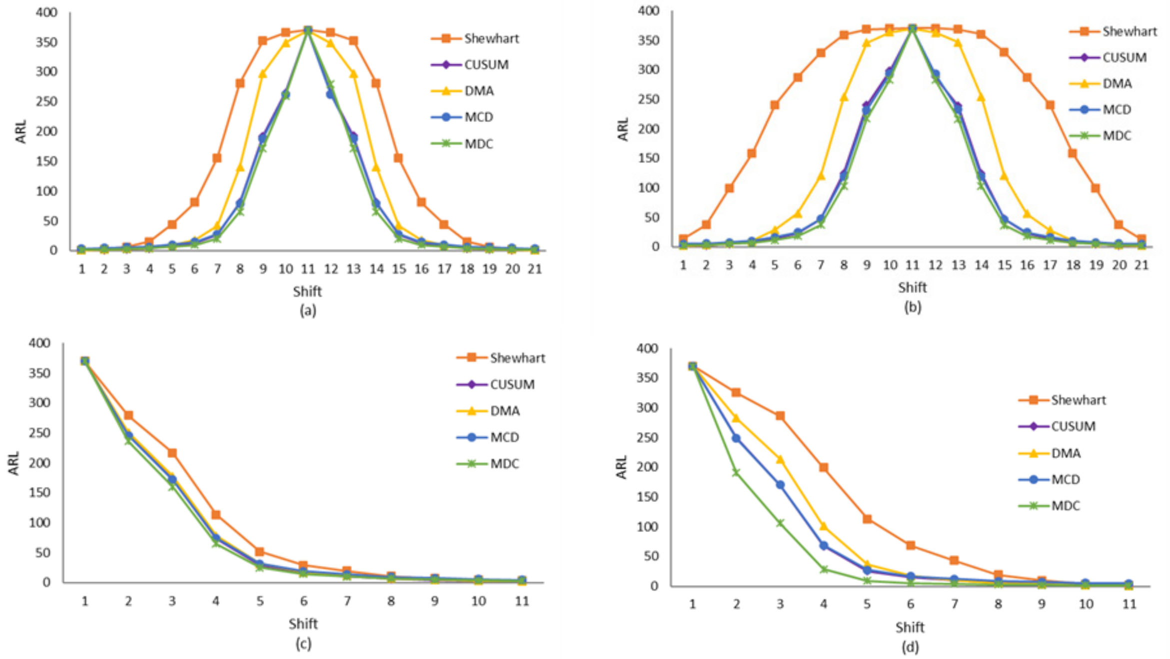

Figure 2: ARL curves of the proposed vs Shewhart, CUSUM, DMA, and MCD control charts: (a) normal distribution, (b) Laplace distribution, (c) exponential distribution, and (d) gamma distribution.

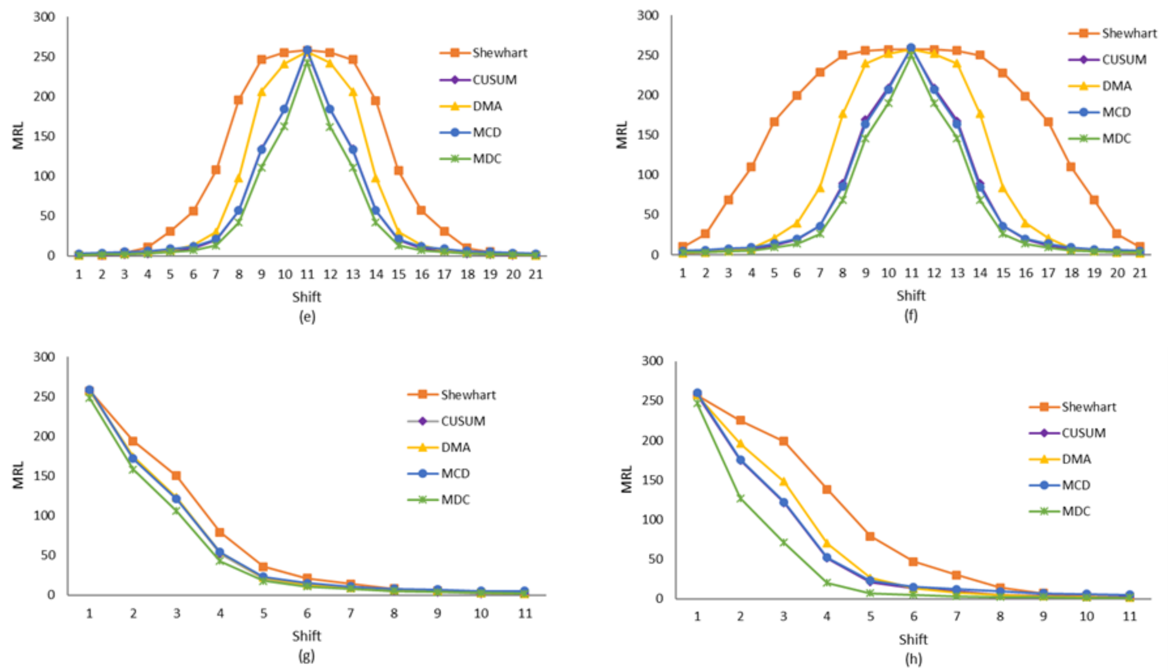

Figure 3: MRL curves of the proposed vs Shewhart, CUSUM, DMA, and MCD control charts: (e) normal distribution, (f) Laplace distribution, (g) exponential distribution, and (h) gamma distribution.

Table 2: ARL and MRL performance of the MDC vs Shewhart, CUSUM, DMA, and MCD control chart for Laplace distribution

δ	Shewhart		CUSUM		DMA		MCD		MDC	
	$H = 2.959$		$H_1 = 7.845$		$H_2 = 3.585$		$H_3 = 15.150$		$H_4 = 20.982$	
	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL
–4.00	13.64	10	1.83	2	2.13	2	5.15	5	2.82	3
–3.00	37.09	26	2.76	3	3.28	3	5.95	6	3.48	3
–2.00	99.26	69	4.94	5	5.82	5	7.83	8	**4.91**	**4**
–1.50	157.95	110	7.55	7	10.39	8	10.10	9	**6.55**	**5**
–1.00	240.60	167	14.32	12	28.76	21	16.28	14	**11.25**	**9**
–0.75	287.13	200	23.42	19	56.47	40	24.76	20	**18.12**	**14**
–0.50	329.12	229	47.50	36	120.58	84	47.10	36	**37.21**	**26**
–0.25	358.92	250	124.78	89	254.22	177	119.25	86	**103.46**	**69**
–0.10	368.69	256	239.93	169	345.60	240	231.90	164	**217.30**	**146**
–0.05	369.84	257	298.24	209	363.71	252	292.73	207	**283.55**	**190**
0.00	370.23	257	370.00	259	370.12	257	370.05	260	370.13	250
0.05	370.48	257	287.86	209	363.19	252	293.13	207	**282.73**	**190**
0.10	368.87	256	238.95	168	346.45	240	232.50	164	**216.87**	**146**
0.25	360.32	250	124.28	89	254.38	177	118.94	85	**103.11**	**69**
0.50	329.99	228	47.39	36	120.57	84	46.97	36	**36.89**	**26**
0.75	287.08	199	23.42	19	56.41	40	24.68	20	**18.04**	**14**
1.00	240.86	167	14.32	12	28.70	21	16.26	14	**11.22**	**9**
1.50	158.22	110	7.55	7	10.37	8	10.10	9	**6.53**	**5**
2.00	98.92	69	4.93	5	5.81	5	7.82	7	**4.90**	**4**
3.00	37.01	26	**2.76**	**3**	3.28	3	5.95	6	3.48	3
4.00	13.59	10	**1.83**	**2**	2.12	2	5.14	5	2.82	3

The minimal values of ARL and MRL are bold.

Table 3: ARL and MRL performance of the MDC vs Shewhart, CUSUM, DMA, and MCD control chart for exponential distribution

δ	Shewhart		CUSUM		DMA		MCD		MDC	
	$H = 4.916$		$H_1 = 6.128$		$H_2 = 5.797$		$H_3 = 5.797$		$H_4 = 20.102$	
	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL
0.00	370.02	257	370.04	257	370.09	257	370.08	259	370.24	248
0.05	279.30	194	247.11	172	250.64	174	245.72	172	236.25	158
0.10	216.46	150	173.53	122	177.71	123	171.93	121	160.06	106
0.25	113.46	79	74.01	52	78.03	54	74.29	54	64.77	43
0.50	51.79	36	29.38	21	31.61	22	31.28	23	25.40	18
0.75	29.30	21	16.49	12	17.69	13	18.95	15	14.74	11
1.00	19.31	14	11.09	9	11.87	9	13.72	11	10.41	8
1.50	10.70	8	**6.58**	**5**	7.08	6	9.34	8	6.80	5
2.00	7.19	5	**4.68**	**4**	5.10	5	7.44	7	5.24	4
3.00	4.38	3	**3.03**	**2**	3.31	3	5.68	5	3.78	3
4.00	3.26	2	**2.32**	**2**	2.44	2	4.77	5	3.05	2

The minimal values of ARL and MRL are bold.

Table 4: ARL and MRL performance of the MDC vs Shewhart, CUSUM, DMA, and MCD control chart for gamma distribution

δ	Shewhart		CUSUM		DMA		MCD		MDC	
	$H = 1.947$		$H_1 = 15.028$		$H_2 = 2.597$		$H_3 = 10.507$		$H_4 = 20.151$	
	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL	ARL	MRL
0.00	370.12	257	370.00	258	370.11	257	370.13	260	370.01	247
0.05	325.40	225	248.89	175	282.86	196	248.46	176	**190.91**	**127**
0.10	286.60	199	170.51	121	213.78	148	170.30	122	**106.44**	**71**
0.25	199.12	138	68.09	51	100.95	70	68.90	52	**28.82**	**20**
0.50	113.33	79	25.72	21	36.75	26	27.73	23	**9.29**	**7**
0.75	68.25	47	14.60	13	17.51	13	17.00	15	**5.52**	**5**
1.00	42.92	30	9.95	9	10.28	8	12.60	12	**4.05**	**3**
1.50	19.41	14	6.58	5	5.24	5	8.88	9	**2.72**	**2**
2.00	10.07	7	4.68	4	3.52	3	7.22	7	**2.08**	**2**
3.00	3.82	3	3.03	2	1.94	2	5.67	6	**1.44**	**1**
4.00	2.06	2	2.32	2	1.14	1	4.94	5	**1.14**	**1**

The minimal values of ARL and MRL are bold.

5 Application Using with Real Data

In this case study; the tensile of single carbon fiber data at 20 mm [19] and the survival times (in days) of guinea pigs infected with virulent *Tubercle bacilli* [20].

5.1 *Application I*

From the data of the tensile of single carbon fiber data at 20 mm. This application consists of 69 observations that were normal distribution. Application I: 0.312, 0.314, 0.479, 0.552, 0.700, 0.803, 0.861, 0.865, 0.944, 0.958, 0.966, 0.997, 1.006, 1.021, 1.027, 1.055, 1.063, 1.098, 1.140, 1.179, 1.224, 1.240, 1.253, 1.270, 1.272, 1.274, 1.301, 1.301, 1.359, 1.382, 1.382, 1.426, 1.434, 1.435, 1.478, 1.490, 1.511, 1.514, 1.535, 1.554, 1.566, 1.570, 1.586, 1.629, 1.633, 1.642, 1.648, 1.684, 1.697, 1.726, 1.770, 1.773, 1.800, 1.809, 1.818, 1.821, 1.848, 1.880, 1.954, 2.012, 2.067, 2.084, 2.090, 2.096, 2.128, 2.233, 2.433, 2.585, 2.585.

The first dataset was applied to the Shewhart, CUSUM, DMA, MCD and MDC charts, as shown in Figure 4. The MCD control chart could detect a change in the process since the 1st observation, the DMA control chart at the 2nd observation, the CUSUM control chart at the 39th observation, the MDC control chart at the 42nd and the Shewhart was unable to detect any change in this dataset.

5.2 *Application II*

This application data of the survival times (in days) of 72 guinea pigs infected with virulent *Tubercle bacilli*. Application II: 12, 15, 22, 24, 24, 32, 32, 33, 34, 38, 38, 43, 44, 48, 52, 53, 54, 54, 55, 56, 57, 58, 58, 59, 60, 60, 60, 60, 61, 62, 63, 65, 65, 67, 68, 70, 70, 72, 73, 75, 76, 76, 81, 83, 84, 85, 87, 91, 95, 96, 98, 99, 109, 110, 121, 127, 129, 131, 143, 146, 146, 175, 175, 211, 233, 258, 258, 263, 297, 341, 341, 376.

This set of data, we applied to all five control charts, as shown n Figure 5. The MCD control chart could detect a change in the process since the 1st observation, the DMA control chart at the 3rd observation, the CUSUM and MCD control charts at the 26th observation and the Shewhart control chart at the 53rd observation.

6 Conclusions

In this research, we present a new control chart called the MDC chart to detect changes in the mean of the process, in cases where symmetric and asymmetric distributions. When the process was in control ARL (ARL_0 = 370), findings illustrated that MDC had better performance than Shewhart, CUSUM, DMA, and MCD charts at all change levels in gamma distribution. While, the normal, Laplace, and exponential distributions showed that MDC had better

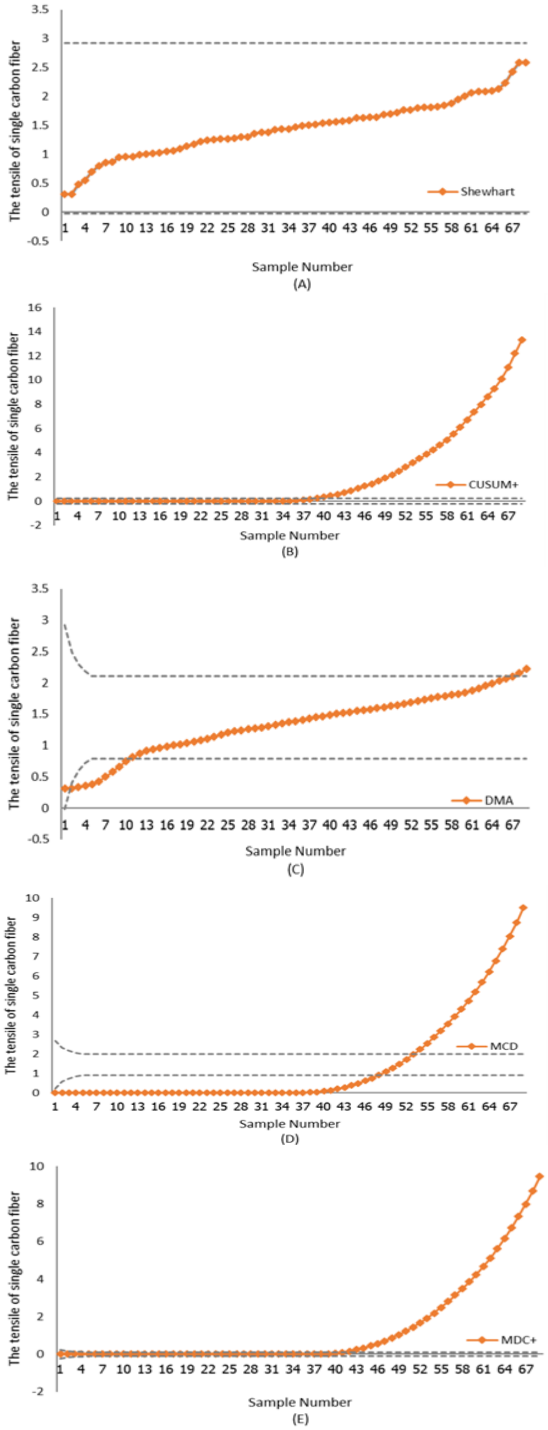

Figure 4: Applying the first dataset to the control charts: (A) Shewhart chart, (B) CUSUM chart, (C) DMA chart, (D) MCD chart, and (E) MDC chart.

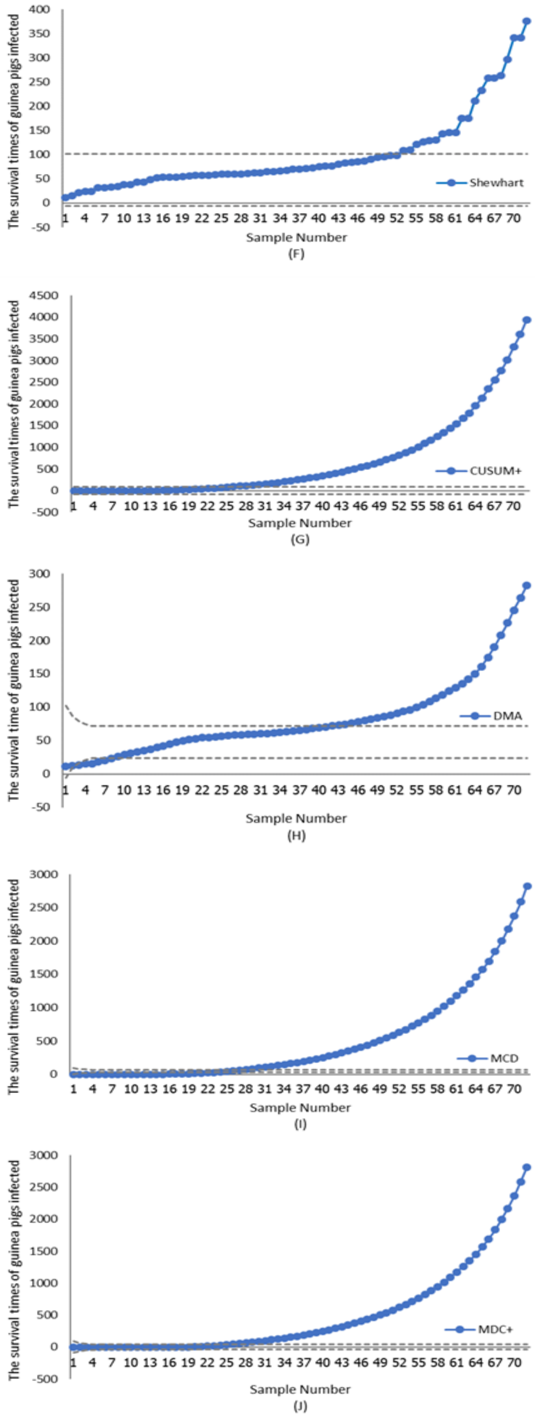

Figure 5: Applying the second dataset to the control charts: (F) Shewhart chart, (G) CUSUM chart, (H) DMA chart, (I) MCD chart, and (J) MDC chart.

performance than the other control charts for small-to-moderate change levels. Furthermore, we compared the performance of the proposed control chart with those of other reported control charts, names MDEWMA [13], MA-CUSUM [14], EWMA-MA [17], and MA-EWMA [21], for a process with normal, Laplace, exponential, and gamma distributed observations. It was found that the MDC control chart performed the best except for large change levels. For research in the future, we will consider the other distributions and design new methods.

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Author Contributions

S.S.: conceptualization, investigation, reviewing and editing; writing—reviewing and editing, funding acquisition, N.S.: investigation, methodology, writing an original draft; S.S. and N.S.: research design, data analysis; Y.A.: conceptualization, data curation, project administration. All authors have read and agreed to the published this version of the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

References

[1] W. A. Shewhart, "Statistical control," in *Economic Control of Quality Manufactured Product*, vol. 11, New York: D. Van Nostrand Company, 1931, p. 145.

[2] E. S. Page, "Continuous inspection schemes," *Biometrika*, vol. 41, no. 1–2, pp. 100–115, Jun. 1954, doi: 10.1093/biomet/41.1-2.100.

[3] B. C. Khoo, "A moving average control chart for monitoring the fraction non-conforming," *Quality and Reliability Engineering International*, vol. 20, no. 6, pp. 617–635, Oct. 2004, doi: 10.1002/qre.576.

[4] B. C. Khoo and V. H. Wong, "A double moving average control chart," *Communications in Statistics – Simulation and Computation*, vol. 37, no. 8, pp. 1696–1708, Oct. 2008, doi: 10.1080/03610910701832459.

[5] B. Zaman, M. Riaz, N. Abbas, and R. J. M. M. Does, "Mixed cumulative sum-exponentially weighted moving average control charts: An efficient way of monitoring process location," *Quality and Reliability Engineering International*, vol. 31, no. 8, pp. 1407–1421, Dec. 2015, doi: 10.1002/qre.1678.

[6] M. Aslam, W. Gui, N. Khan, and C. -H. Jun, "Double moving average-EWMA control chart for exponentially distributed quality," *Communications in Statistics – Simulation and Computation*, vol. 46, no. 9, pp. 7351–7364, Apr. 2017, doi: 10.1080/03610918.2016.1236955.

[7] J. O. Ajadi and M. Riaz, "Mixed multivariate EWMA-CUSUM control charts for an improved process monitoring," *Communications in Statistics – Theory and Methods*, vol. 46, no. 14, pp. 6980–6993, Mar. 2017, doi: 10.1080/03610926.2016.1139132.

[8] N. Abbas, I. A. Raji, M. Riaz, and K. A. L. -Ghamdi, "On designing mixed EWMA Dual-CUSUM chart with applications in petro-chemical industry," *IEEE Access*, vol. 6, pp. 78931–78946, Dec. 2018, doi: 10.1109/ ACCESS.2018.2885598.

[9] R. Thitisoowaranon, S. Sukparungsee, and Y. Areepong, "A mixed cumulative sum-Tukey's control chart for detecting process dispersion," *The Journal of KMUTNB*, vol. 29, no. 3, pp. 507–517, Jul.-Sep. 2019, doi: 10.14416/j.kmutnb.2019.04.004.

[10] C. C. Alves, A. C. Konrath, E. Henning, O. M. F. C. Walter, E. P. Paladini, T. A. Oliveria, and A. Oliveira, "The mixed CUSUM-EWMA (MCE) control chart as a new alternative in the monitoring of a manufacturing process," *Brazillian Journal of Operations & Production Management*, vol. 6, no. 1, pp. 1–13, Mar. 2019, doi: 10.14488/BJOPM.2019.v16.n1.a1.

[11] S. Hussain, X. Wang, S. Ahmand, and M. Riaz, "On a class of mixed EWMA-CUSUM median control charts for process monitoring," *Quality*

and Reliability Engineering International, vol. 36, no. 3, pp. 910–946, Apr. 2020, doi: 10.1002/qre.2608.
[12] M. Abid, S. Mei, H. Z. Nazir, and M. Riaz, "A mixed HWMA-CUSUM mean chart with an application to manufacturing process," *Quality and Reliability Engineering International*, vol. 37, no. 2, pp. 618–631, Mar. 2021, doi: 10.1002/qre.2752.
[13] S. Phantu and S. Sukparungsee, "A mixed double exponentially weighted moving average-Tukey's control chart for monitoring of parameter change," *Thailand Statistician*, vol. 18, no. 4, pp. 392–402, Oct. 2020.
[14] N. Saengsura, S. Sukparungsee, and Y. Areepong, "Mixed moving average-cumulative sum control chart for monitoring parameter change," *Intelligent Automation & Soft Computing*, vol. 31, no. 1, pp. 635–647, 2022, doi: 10.32604/iasc.2022.019997.
[15] S. Sukparungsee, N. Saengsura, Y. Areepong, and S. Phantu, "Mixed Tukey-double moving average for monitoring of process mean," *Thailand Statistician*, vol. 19, no. 4, pp. 885–865, Oct. 2021.
[16] D. C. Montgomery, *Introduction to Statistical Quality Control Case Study*, 6th ed. New York: John Wiley and Sons, 2009.
[17] S. Sukparungsee, Y. Areepong, and R. Taboran, "Exponentially weighted moving average-moving average charts for monitoring the process mean," *PLOS ONE*, vol. 15, no. 2, 2020, doi: 10.1371/journal.pone.0228208.
[18] F. F. Gan, "An optimal design of cumulative sum control charts based on median run length," *Communications in Statistics - Simulation and Computation*, vol. 23, no. 2, pp. 485–503, 1994, doi: 10.1080/03610919408813183.
[19] R. Taboran, S. Sukparungsee, and Y. Areepong, "Design of a new Tukey MA-DEWMA control chart to monitor process and its applications," *IEEE Access*, vol. 9, pp. 102746–102757, Jul. 2021, doi: 10.1109/ACCESS.2021.3098172.
[20] B. Efron, "Logistic regression survival analysis and the Kaplan-Meier curve," *Journal of the American Statistical Association*, vol. 83, no. 402, pp. 414–425, 1988, doi: 10.2307/2288857.
[21] R. Taboran, S. Sukparungsee, and Y. Areepong, "Mixed moving average-exponentially weighted moving average control charts for monitoring of parameter change," in *International MultiConference of Engineers and Computer Scientists*, 2019, pp. 411–415.