Research Article

On Designing a Moving Average-Range Control Chart for Enhancing a Process Variation Detection

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Abstract
This research purpose is to create a moving average control chart for detecting a change in process variations with a range so-called *MA-R* chart and to compare the performance of the *MA-R* chart with the *R*, *S*, and *MA-S* control charts for detecting variation changes. The purposed control chart is an effective alternative to the R control chart using the moving average based on the sample range. The coefficients for the control limit of *MA-R* varying the sample sizes (m) and the width for moving average calculation (w) are presented. Comparison and application to real data show that the *MA-R* control chart is more effective at detecting variations at all levels than the *R* and *S* control charts. Furthermore, when the magnitude of the variation is small, the *MA-R* chart becomes more effective as w increases.

Keywords: Time-varying chart, Variations, Efficiency, Monitoring, Average run length

1 Introduction

Nowadays, quality control in industrial production processes plays a vital role and is important. To be the same standard for both production processes and quality, making consumers have credibility and acceptance of the product. In practice, however, the production process may have variations that affect the quality of the product. The factors that cause variation in the production process are machinery, workers, management, as well as raw materials. Industrial production processes use statistical tools to control production quality to make the product quality meets the standards. The most widely used tool is the "control chart" because it is a powerful tool and can be displayed as a graphical results when the production process changes. Control charts can be divided into 2 types: control charts for variables are control charts used when measuring product quality from weight and measure, which is quantitative data and is a continuous value such as weight, diameter and lifetime, etc. Examples of variable control charts, such as average control chart (*x-bar* chart), and range control chart (*R* chart). The second type is control charts for attributes, which are used for detecting the number of defects or the number of nonconformities which is counting data and is an integer. An example of this type of control chart is the defect proportion control chart (*p* chart), the number of defect control chart (*np* chart), and the number of nonconforming products per unit control chart (*u* chart), etc.

In 1924, Shewhart [1] proposed the Shewhart control chart, which is a powerful control chart for detecting large average changes, however, cannot detect small mean shifts. Subsequently, other quality control charts were developed that were more effective at detecting small changes than the Shewhart control chart. For example, in 1959 Roberts [2] presented a weighted moving average control namely an Exponentially Weighted Moving Average Control Chart (*EWMA*) using the principle of taking data over time of the observations in the collection process make decisions. It was found that the *EWMA* control chart outperformed the Shewhart control chart when the magnitude of the change in the process mean was small. Later in 2004, Khoo [3] developed a Moving Average control chart (*MA*) using a simple idea to calculate the MA statistics by giving a width of average (w). This control chart is easy to calculate and implement as well as its efficiency suit for small to moderate shifts (see Areepong and Sukparungsee [4], Chananet *et al.*, [5], Raweesawat and Sukparungsee [6]). Control charts are commonly used with mean values along with measures of variation such as range and standard deviation. Process variability measures are more important than process mean in some situations i.e., monitoring the consistency of the process. Therefore, process variation control charts need to be developed to bring the process back as smoothly as possible.

In 2016, Olatunde and Olaomia [7] developed an *MA* control chart for the standard deviation. It is called the *MA-S* control chart and proposed the explicit formulas to determine the average run length (ARL) and compare the results in detecting variance changes with the *S* chart (see also Phantu and Sukparungsee [8] and Sukparungse *et al.*, [9]). Later, in 2019, Olatunde *et al.*, [10] proposed a *DMA-S* control chart that enhanced the ability to detect changes in process variability. Along with presenting a successful formula for finding the ARL and comparing the performance of the variance change with *S* and *MA-S* charts. The performance of *DMA-S* control charts outperforms other control charts, which is suitable for detecting small to moderate variance changes in the process when the process has a normal distribution (see also Sukparungsee *et al.*, [11]). In addition, there is also a control chart for the variation, the range control chart (*R* chart), that is easy to calculate and works well when the sample size is small (n <10). In this research, the *MA* control chart is developed to create a new control chart for the detection of a change in variation based on range, namely the *MA-R* control chart. In addition, the performance of the *MA-R* control chart is compared with the *MA-S* control chart for detecting process variations and applying them to real data the control chart gives the lowest value ARL_1 indicating that the control chart is most effective in detecting variation changes.

2 Designing of Control Charts and Properties

In this research, a new control chart named "moving average of range (*MA-R*) chart" for detecting process variability is investigated, and the statistic of control charts and the control limits are presented. The performance of the proposed control chart is compared with the performance of an *MA-S* control chart. Generally, the *R* chart is more popular among quality control practitioners especially when dealing with small sample sizes because of the simplicity of calculating the range from each sample. Therefore, in this section, the study control charts and related research are discussed as follows.

2.1 *Range control chart (R chart)*

A range chart is a statistical process control (SPC) tool that displays the variation within a set of data. It is used to track the variation in a process over time and helps identify any changes in the process variance. It plots the range of the data in each subgroup where the range is calculated from the difference between the highest and lowest values in each subgroup over time. The *R* chart is suitable if the sample sizes (n) are small ($n \leq 10$). For developing a quality control chart, it is essential to always consider this *R* chart in conjunction with the *x-bar* chart which can be calculated to find the average of the range ($\bar{R}$) as follows:

$$\bar{R} = \frac{\sum_{j=1}^{m} R_j}{m}$$

where R_j is the difference between the highest value in sample j and the lowest value in sample j).

The calculation of the upper control limit (UCL) and lower control limit (LCL) is divided into 2 cases; known and unknown parameters σ. For the latter case, the parameter must be estimated Montgomery [12] stated that in the process variability, an unbiased estimator of σ, is $\hat{\sigma} = \frac{\bar{R}}{d_2}$ for *R* chart and is $\hat{\sigma} = \frac{\bar{S}}{C_4}$ for *S* chart, respectively. Consequently, the control limits are as follows:

1) Known σ

$$\begin{aligned} UCL &= d_2\sigma + 3d_3\sigma = D_2\sigma \\ CL &= d_2\sigma \\ LCL &= d_2\sigma - 3d_3\sigma = D_1\sigma \end{aligned} \tag{1}$$

where the values from Equation (1), $D_1 = (d_2 - 3d_3)$ and $D_2 = (d_2 + 3d_3)$, are factors for control limits and depend on the sample size (n). The tables of the factor of control limits are addressed in several quality control books (i.e., [12]).

2) Unknown σ, then an estimate $\hat{\sigma} = \dfrac{\overline{R}}{d_2}$

$$\begin{aligned} UCL &= \overline{R} + 3\frac{d_3}{d_2}\overline{R} = D_4\overline{R} \\ CL &= \overline{R} \\ LCL &= \overline{R} - 3\frac{d_3}{d_2}\overline{R} = D_3\overline{R}. \end{aligned} \tag{2}$$

Then the values from Equation (2), $D_3 = \left(1 - 3\dfrac{d_3}{d_2}\right)$ and $D_4 = \left(1 + 3\dfrac{d_3}{d_2}\right)$. In addition, their values are a constant found in the factor of control limits as well as D_1 and D_2.

2.2 *Standard deviation control chart (S chart)*

The standard deviation control chart (S chart) is a chart commonly used in cases where the sample size (n) of a subgroup is greater than 10 (n >10). Estimating the population standard deviation ($\hat{\sigma}$) by the sample standard deviation (S) is more informative than the range (R). Therefore, in the case of large sample sizes, the S chart provides more information than the R chart when the standard deviation of the sample can be calculated as follows:

$$S = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}}. \tag{3}$$

The mean of the standard deviation can be calculated as

$$\overline{S} = \frac{\sum_{j=1}^{m} S_j}{m}. \tag{4}$$

The calculation of the upper control limit (UCL) and lower control limit (LCL) is divided into 2 cases:

1) Known σ

$$\begin{aligned} UCL &= C_4\sigma + 3\sigma\sqrt{(1 - C_4^2)} = B_6\sigma \\ CL &= C_4\sigma \\ LCL &= C_4\sigma - 3\sigma\sqrt{(1 - C_4^2)} = B_5\sigma, \end{aligned} \tag{5}$$

where the values from Equation (5) are constant, $B_5 = C_4 - 3\sqrt{1 - C_4^2}$ and $B_6 = C_4 + 3\sqrt{1 - C_4^2}$.

2) Unknown σ, then an estimate $\hat{\sigma} = \dfrac{\overline{S}}{C_4}$

$$\begin{aligned} UCL &= \overline{S} + 3\frac{\overline{S}}{C_4}\sqrt{1 - C_4^2} = B_4\overline{S} \\ CL &= \overline{S} \\ LCL &= \overline{S} - 3\frac{\overline{S}}{C_4}\sqrt{1 - C_4^2} = B_3\overline{S}, \end{aligned} \tag{6}$$

where the values from Equation (6) are constant, $B_3 = \left(1 - \dfrac{3\sqrt{1 - C_4^2}}{C_4}\right)$ and $B_4 = \left(1 + \dfrac{3\sqrt{1 - C_4^2}}{C_4}\right)$.

2.3 *Moving average - standard deviation control chart (MA-S chart)*

The moving average control chart based on standard deviation (*MA-S*) is an application of the moving average control chart to detect the process variability with the standard deviation value proposed by Olatunde and Olaomia [7]. Factors for the *MA-S* chart based on sample sizes (n > 2) were presented and the performance of the *MA-S* chart was compared with the S control chart. The results show that the *MA-S* chart performance at w = 2, 3, and 4 is more effective than the S control chart for simulated data. The standard deviation (S) and mean sample standard deviation ($\overline{S}$) can be calculated from Equations (3) and (4), respectively. The MA statistics for the standard deviation of width (w) at times i are calculated as

$$MA - S_i = \begin{cases} \dfrac{S_i + S_{i-1} + S_{i-2} + \ldots + S_1}{i} & ; i < w \\ \dfrac{S_i + S_{i-1} + \ldots + S_{i-w+1}}{w} & ; i \geq w \end{cases}. \tag{7}$$

The *MA-S* statistics from Equation (7) can be rewritten as follows

$$MA-S_i = \begin{cases} \dfrac{\sum_{j=1}^{i} S_j}{i} & ; i < w \\ \dfrac{\sum_{j=i-w+1}^{i} S_j}{w} & ; i \geq w \end{cases}.$$

The expectation of the *MA-S* chart for both cases $i < w$ and $i \geq w$ is presented in Equations (8),

$$E(MA-S) = C_4\sigma. \quad (8)$$

The variance of the *MA-S* chart when $i < w$ and $i \geq w$ are shown in Equations (9) as follows:

$$Var(MA-S) = \begin{cases} \dfrac{\sigma^2(1-C_4^2)}{i} & ; i < w \\ \dfrac{\sigma^2(1-C_4^2)}{w} & ; i \geq w \end{cases}. \quad (9)$$

The calculation of the upper control limit (UCL) and lower control limit (LCL) of the *MA-S* chart is divided into 2 cases as follows:

1) Known σ

1.1) when $i < w$,

$$UCL = C_4\sigma + 3\sigma\sqrt{\frac{(1-C_4^2)}{i}} = B_8^*\sigma$$
$$CL = C_4\sigma$$
$$LCL = C_4\sigma - 3\sigma\sqrt{\frac{(1-C_4^2)}{i}} = B_7^*\sigma,$$

where
$B_7^* = (C_4 - 3\sqrt{(1-C_4^2)/i})$, and $B_8^* = (C_4 + 3\sqrt{(1-C_4^2)/i})$.

1.2) when $i \geq w$,

$$UCL = C_4\sigma + 3\sigma\sqrt{\frac{(1-C_4^2)}{w}} = B_{10}^*\sigma$$
$$CL = C_4\sigma$$
$$LCL = C_4\sigma - 3\sigma\sqrt{\frac{(1-C_4^2)}{w}} = B_9^*\sigma,$$

where
$B_9^* = (C_4 - 3\sqrt{(1-C_4^2)/w})$, and $B_{10}^* = (C_4 + 3\sqrt{(1-C_4^2)/w})$.

2) Unknown σ

2.1) when $i < w$,

$$UCL = \bar{S} + 3\frac{\bar{S}}{C_4}\sqrt{\frac{1-C_4^2}{i}} = B_{12}^*\bar{S}$$
$$CL = \bar{S}$$
$$LCL = \bar{S} - 3\frac{\bar{S}}{C_4}\sqrt{\frac{1-C_4^2}{i}} = B_{11}^*\bar{S},$$

where
$$B_{11}^* = \left(1 - 3\sqrt{\frac{1-C_4^2}{i}}\Big/C_4\right), \text{ and } B_{12}^* = \left(1 + 3\sqrt{\frac{1-C_4^2}{i}}\Big/C_4\right).$$

2.2) when $i \geq w$,

$$UCL = \bar{S} + 3\frac{\bar{S}}{C_4}\sqrt{\frac{1-C_4^2}{w}} = B_{14}^*\bar{S}$$
$$CL = \bar{S}$$
$$LCL = \bar{S} - 3\frac{\bar{S}}{C_4}\sqrt{\frac{1-C_4^2}{w}} = B_{13}^*\bar{S},$$

where
$$B_{13}^* = \left(1 - 3\sqrt{\frac{1-C_4^2}{w}}\Big/C_4\right), \text{ and } B_{14}^* = \left(1 + 3\sqrt{\frac{1-C_4^2}{w}}\Big/C_4\right).$$

Besides, the factor of *MA-S* control limits B_{11}^*, B_{12}^*, B_{13}^* and B_{14}^* is similar to the values of D_5^*, D_6^*, D_7^* and D_8^*, which proposed by Olatunde and Olaomia [7], respectively.

2.4 *The proposed control chart (MA-R chart)*

The moving average control chart (MA) can be used to detect a change in the process mean as well as process variability. In this paper, the *MA* chart is implemented to detect a change in process variation based on the range value instead the standard deviation which Olatunde and Olaomia [7]) proposed in the *MA-S* control chart. This modified *MA* chart with the range value to monitor changes in process variation, namely the Moving Average– Range control chart (*MA-R* chart). This research aims to construct a new control chart and the factor of control limits table which depends on the sample size (n). The *MA-R* statistic of width w at times i is calculated as

$$MA-R_i = \begin{cases} \dfrac{R_i + R_{i-1} + R_{i-2} + \ldots + R_1}{i} & ; i < w \\ \dfrac{R_i + R_{i-1} + \ldots + R_{i-w+1}}{w} & ; i \geq w, \end{cases} \tag{10}$$

where R_j is the range of each sample number. The *MA-R* statistics from Equation (10) can be rewritten as follows

$$MA-R_i = \begin{cases} \dfrac{\sum_{j=1}^{i} R_j}{i} & ; i < w \\ \dfrac{\sum_{j=i-w+1}^{i} R_j}{w} & ; i \geq w \end{cases}. \tag{11}$$

The expectation of the *MA-R* chart when $i < w$, is presented in Equation (12),

$$\begin{aligned} E(MA-R) &= E\left(\frac{1}{i}\sum_{j=1}^{i} R_j\right) \\ &= \frac{1}{i}\sum_{j=1}^{i} E(R_j) = d_2\sigma. \end{aligned} \tag{12}$$

Also, the expectation of the *MA-R* chart when $i \geq w$, shown in Equations (13),

$$\begin{aligned} E(MA-R) &= E\left(\frac{1}{w}\sum_{j=i-w+1}^{w} R_j\right) \\ &= \frac{1}{w}\sum_{j=i-w+1}^{w} E(R_j) = d_2\sigma. \end{aligned} \tag{13}$$

The variance of the *MA-R* chart when $i < w$, is presented in Equations (14),

$$\begin{aligned} Var(MA-R) &= Var\left(\frac{1}{i}\sum_{j=1}^{i} R_j\right) \\ &= \frac{1}{i^2}\sum_{j=1}^{i} Var(R_j) = \frac{d_3^2\sigma^2}{i}. \end{aligned} \tag{14}$$

Also, the variance of the *MA-R* chart when $i \geq w$, shown in Equations (15),

$$\begin{aligned} Var(MA-R) &= Var\left(\frac{1}{w}\sum_{j=i-w+1}^{i} R_j\right) \\ &= \frac{1}{w^2}\sum_{j=i-w+1}^{i} Var(R_j) = \frac{d_3^2\sigma^2}{w}. \end{aligned} \tag{15}$$

Therefore, the upper control limit (UCL) and lower control limit (LCL) of the *MA-R* chart can be calculated in 2 cases following:

1) Known σ

1.1) when $i < w$,

$$\begin{aligned} UCL &= d_2\sigma + 3\sqrt{\frac{d_3^2\sigma^2}{i}} = d_2\sigma + 3d_3\sigma\sqrt{\frac{1}{i}} \\ CL &= d_2\sigma \\ LCL &= d_2\sigma - 3\sqrt{\frac{d_3^2\sigma^2}{i}} = d_2\sigma - 3d_3\sigma\sqrt{\frac{1}{i}}. \end{aligned} \tag{16}$$

The Equation (16) can be rewritten as follows:

$$\begin{aligned} UCL &= D_6^*\sigma \\ CL &= d_2\sigma \\ LCL &= D_5^*\sigma, \end{aligned} \tag{17}$$

where $D_5^* = \left(d_2 - 3d_3\sqrt{\frac{1}{i}}\right)$ and $D_6^* = \left(d_2 + 3d_3\sqrt{\frac{1}{i}}\right)$, they are the factor of control limits which are calculated and proposed in the next section.

1.2) when $i \geq w$,

$$\begin{aligned} UCL &= d_2\sigma + 3\sqrt{\frac{d_3^2\sigma^2}{w}} = d_2\sigma + 3d_3\sigma\sqrt{\frac{1}{w}} \\ CL &= d_2\sigma \\ LCL &= d_2\sigma - 3\sqrt{\frac{d_3^2\sigma^2}{w}} = d_2\sigma - 3d_3\sigma\sqrt{\frac{1}{w}}. \end{aligned} \tag{18}$$

The Equation (18) can be rewritten as follows:

$$\begin{aligned} UCL &= D_8^*\sigma \\ CL &= d_2\sigma \\ LCL &= D_7^*\sigma, \end{aligned} \tag{19}$$

where $D_7^* = \left(d_2 - 3d_3\sqrt{\frac{1}{w}}\right)$ and $D_8^* = \left(d_2 + 3d_3\sqrt{\frac{1}{w}}\right)$, they are the factor of control limits from the proposed chart.

2) Unknown σ

2.1) when $i < w$,

$$\begin{aligned} UCL &= \bar{R} + 3\bar{R}\frac{d_3}{d_2}\sqrt{\frac{1}{i}} \\ CL &= d_2\sigma \\ LCL &= \bar{R} - 3\bar{R}\frac{d_3}{d_2}\sqrt{\frac{1}{i}}. \end{aligned} \tag{20}$$

The Equation (20) can be rewritten as follows:

$$UCL = D_{10}^{*}\bar{R}$$
$$CL \;= d_2\sigma$$
$$LCL = D_9^{*}\bar{R}, \tag{21}$$

where $D_9^{*}=\left(1-3\frac{d_3}{d_2}\sqrt{\frac{1}{i}}\right)$ and $D_{10}^{*}=\left(1+3\frac{d_3}{d_2}\sqrt{\frac{1}{i}}\right)$.

2.1) when $i \geq w$,

$$UCL = \bar{R}+3\bar{R}\frac{d_3}{d_2}\sqrt{\frac{1}{w}}$$
$$CL \;= \bar{R}$$
$$LCL = \bar{R}-3\bar{R}\frac{d_3}{d_2}\sqrt{\frac{1}{w}}. \tag{22}$$

The Equation (22) can be rewritten as follows:

$$UCL = D_{12}^{*}\bar{R}$$
$$CL \;= \bar{R}$$
$$LCL = D_{11}^{*}\bar{R}, \tag{23}$$

where $D_{11}^{*}=\left(1-3\frac{d_3}{d_2}\sqrt{\frac{1}{w}}\right)$ and $D_{12}^{*}=\left(1+3\frac{d_3}{d_2}\sqrt{\frac{1}{w}}\right)$.

3 Results

In this section, the factor of control limits for each case is calculated as shown in Tables 1–4. Table 1 and Table 2 show the coefficient of control limits of the *MA-R* chart for known parameters σ and w = 5, 10, 15, and 20 with given w = 5 when $i < w$, and $i \geq w$ calculated from Equations (17) and (19), respectively. Next, the coefficient of control limits of the *MA-R* chart for unknown parameter σ, s with given w = 5 when $i < w$, $i \geq w$ calculated from Equations (21) and (23), are presented in Tables 3 and 4, respectively. Therefore, the coefficient of control limits tables (see Tables 1–4) are very useful and prompt for practitioners. In addition, the applications of the *MA-R* chart are demonstrated for two examples; 1) the simulated data and 2) real data as flow width measurements (microns) for the hard-bake process from [12].

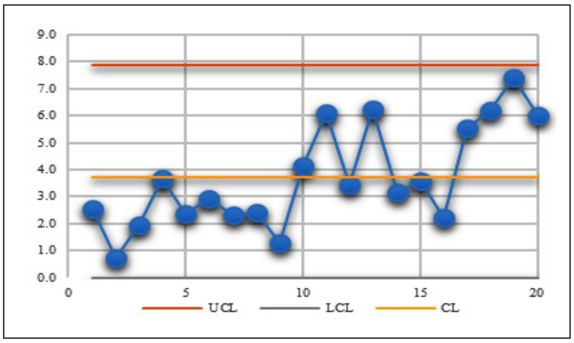
3.1 *Application I: The simulated data*

We used the simulated data from the example of [7] which assumed that the observations are from Normal (10, 1) distribution when the process is in-control for 10 subgroups. The process variability is changed two times of standard deviation to be Normal (10, 2) for the next 10 subgroups and given sample size (n) = 5. In addition, the simulated data for 20 subgroups are calculated with the value of range and standard deviation presented and applied with a moving average chart. Therefore, the performance of detection of a change in process variability of the proposed control chart with width w = 2, 3, 4, 5, 10, 15 and 20 is compared with the performance of the *R* chart shown in Table 5 and the control limits of those control charts are calculated from the Equation [21] and [23] which addressed on Table 6. The control limits of the *S* chart and *MA-S* charts are referred from [7]. The performance comparison of *R*, *MA-R*, *S*, and *MA-S* control charts are presented as a graphical display in Figure 1.

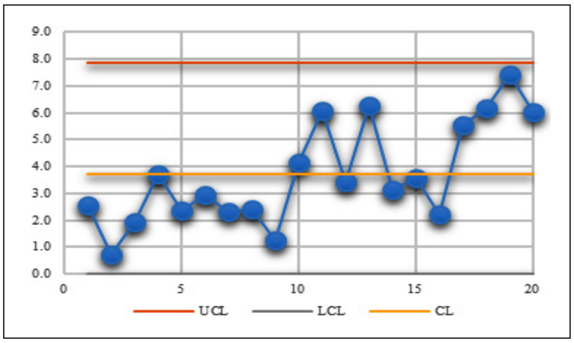
From Figure 1, the performance of the *R* chart, *S* chart, *MA-S*, and *MA-R* charts for width w = 2, 3, and 4 are shown and the change needs to be quickly detected. The results found that the performance of *R* and *S* charts could not detect a process variability as shown in Figure 1(a) and (b), respectively. The *MA-R* and *MA-S* with width w = 3 outperform other charts which can quickly detect. Furthermore, the efficiency of the *MA-R* and *MA-S* are in excellent agreement to detect the process variability, however, the *MA-R* chart based on the range value is simple to calculate compared with the *MA-S* based on the standard deviation value.

3.2 *Application II: The flow width measurements in the hard-bake process*

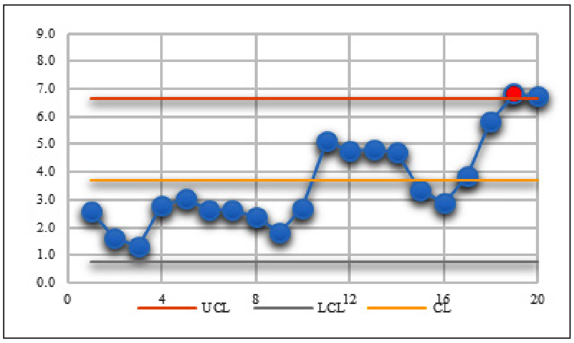
This example consists of 25 observations, each of size five wafers. The control limits of the *R* and *MA-R* charts calculate from Equations (2) and (23), respectively. Consequently, the performance of detection of a change in process variability of the proposed control chart with width w = 2, 3, 4, 5, 10, 15 and 20 is compared with the performance of the *R* chart shown in Table 7, and the control limits of *S* chart and *MA-S* charts is calculated as Table 8. The performance comparison of R, *MA-R*, *S*, and *MA-S* charts is presented as a graphical display in Figure 2.



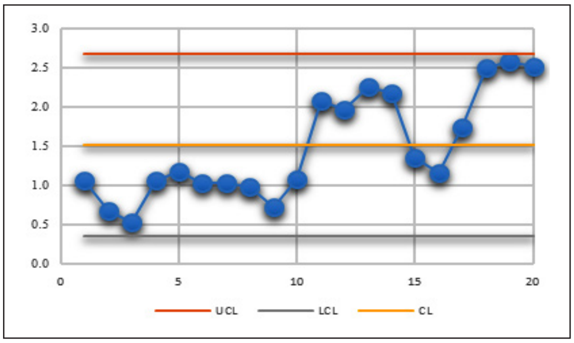
(a) *R* chart



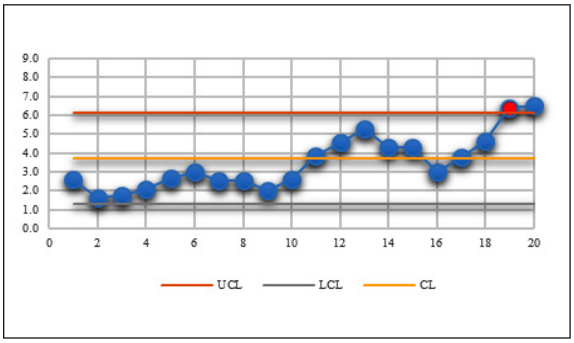
(b) *S* chart



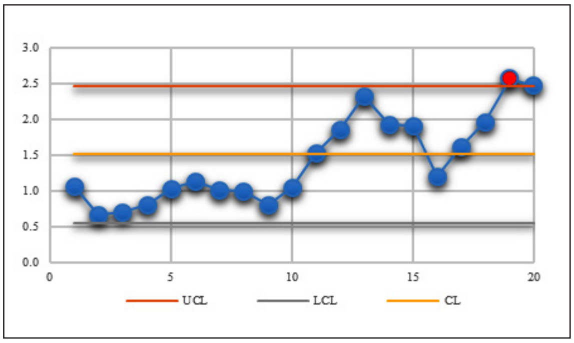
(c) *MA-R* chart with $w = 2$



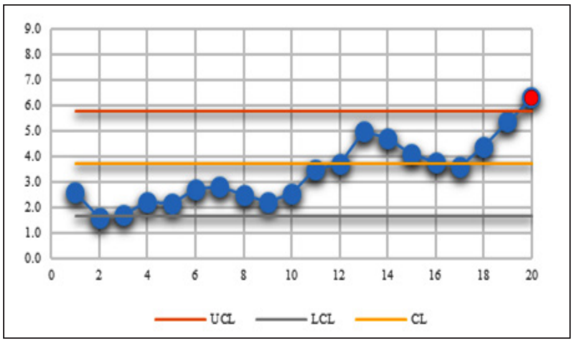
(d) *MA-S* chart with $w = 2$



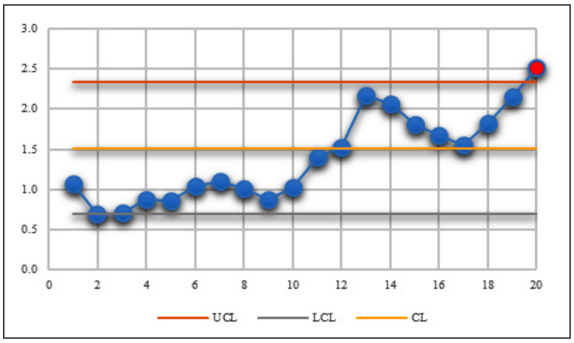
(e) *MA-R* chart with $w = 3$



(f) *MA-S* chart with $w = 3$

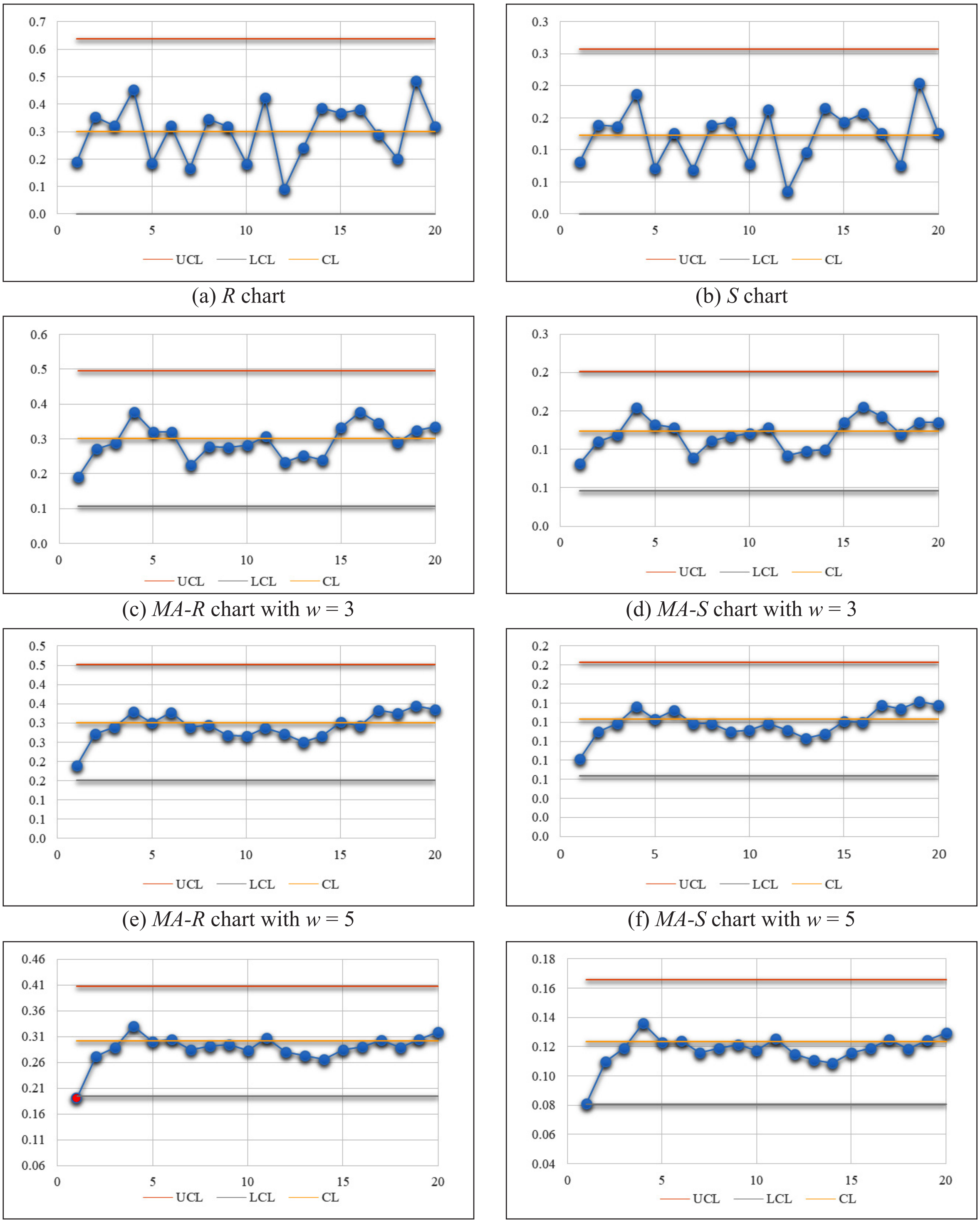


(g) *MA-R* chart with $w = 4$



(h) *MA-S* chart with $w = 4$

Figure 1: Performance of control charts from simulated data.



(a) *R* chart (b) *S* chart

(c) *MA-R* chart with $w = 3$ (d) *MA-S* chart with $w = 3$

(e) *MA-R* chart with $w = 5$ (f) *MA-S* chart with $w = 5$

(g) *MA-R* chart with $w = 10$ (h) *MA-S* chart with $w = 10$

Figure 2: Performance of control charts from flow width measurements in hard-bake process.

From Figure 2, the performance of the *R* chart, *S* chart, *MA-S*, and *MA-R* charts for width w = 3, 5, and 10 are compared which needs to bring the process back to normal. The performance of the *R* and *S* chart is poor to detect a change in process variability. In addition, the proposed control chart; the *MA-R* chart outperforms the *MA-S* chart which can detect an early change in process variability when w = 10. It found that when the magnitudes of the change in the process variation become smaller the moving average control chart performs better as w increases (Figure 2).

Table 1: A coefficient of *MA-R* control limits for the known parameter σ when $i < w$

Sample Size (n)	d_2	d_3	$i = 1$		$i = 2$		$i = 3$		$i = 4$	
			D_5^*	D_6^*	D_5^*	D_6^*	D_5^*	D_6^*	D_5^*	D_6^*
2	1.128	0.853	0	3.687	0	2.937	0	2.605	0	2.408
3	1.693	0.888	0	4.357	0	3.577	0.155	3.231	0.361	3.025
4	2.059	0.880	0	4.699	0.192	3.926	0.535	3.583	0.739	3.379
5	2.326	0.864	0	4.918	0.493	4.159	0.830	3.822	1.030	3.622
6	2.534	0.848	0	5.078	0.735	4.333	1.065	4.003	1.262	3.806
7	2.704	0.833	0.205	5.203	0.937	4.471	1.261	4.147	1.455	3.954
8	2.847	0.820	0.387	5.307	1.108	4.586	1.427	4.267	1.617	4.077
9	2.970	0.808	0.546	5.394	1.256	4.684	1.571	4.369	1.758	4.182
10	3.078	0.797	0.687	5.469	1.387	4.769	1.698	4.458	1.883	4.274
11	3.173	0.787	0.812	5.534	1.504	4.842	1.810	4.536	1.993	4.354
12	3.258	0.778	0.924	5.592	1.608	4.908	1.910	4.606	2.091	4.425
13	3.336	0.770	1.026	5.646	1.703	4.969	2.002	4.670	2.181	4.491
14	3.407	0.762	1.121	5.693	1.791	5.023	2.087	4.727	2.264	4.550
15	3.472	0.755	1.207	5.737	1.870	5.074	2.164	4.780	2.340	4.605
16	3.532	0.749	1.285	5.779	1.943	5.121	2.235	4.829	2.409	4.656
17	3.588	0.743	1.359	5.817	2.012	5.164	2.301	4.875	2.474	4.703
18	3.640	0.738	1.426	5.854	2.074	5.206	2.362	4.918	2.533	4.747
19	3.689	0.733	1.490	5.888	2.134	5.244	2.419	4.959	2.590	4.789
20	3.735	0.729	1.548	5.922	2.189	5.281	2.472	4.998	2.642	4.829
21	3.778	0.724	1.606	5.950	2.242	5.314	2.524	5.032	2.692	4.864
22	3.819	0.720	1.659	5.979	2.292	5.346	2.572	5.066	2.739	4.899
23	3.858	0.716	1.710	6.006	2.339	5.377	2.618	5.098	2.784	4.932
24	3.895	0.712	1.759	6.031	2.385	5.405	2.662	5.128	2.827	4.963
25	3.931	0.709	1.804	6.058	2.427	5.435	2.703	5.159	2.868	4.995

Table 2: A coefficient of *MA-R* control limits for known parameter σ, s when $i \geq w$

Sample Size (n)	d_2	d_3	$w = 5$		$w = 10$		$w = 15$		$w = 20$	
			D_7^*	D_8^*	D_7^*	D_8^*	D_7^*	D_8^*	D_7^*	D_8^*
2	1.128	0.853	0	2.272	0.319	1.937	0.467	1.789	0.556	1.700
3	1.693	0.888	0.502	2.884	0.851	2.535	1.005	2.381	1.097	2.289
4	2.059	0.880	0.878	3.240	1.224	2.894	1.377	2.741	1.469	2.649
5	2.326	0.864	1.167	3.485	1.506	3.146	1.657	2.995	1.746	2.906
6	2.534	0.848	1.396	3.672	1.730	3.338	1.877	3.191	1.965	3.103
7	2.704	0.833	1.586	3.822	1.914	3.494	2.059	3.349	2.145	3.263
8	2.847	0.820	1.747	3.947	2.069	3.625	2.212	3.482	2.297	3.397
9	2.970	0.808	1.886	4.054	2.203	3.737	2.344	3.596	2.428	3.512
10	3.078	0.797	2.009	4.147	2.322	3.834	2.461	3.695	2.543	3.613
11	3.173	0.787	2.117	4.229	2.426	3.920	2.563	3.783	2.645	3.701
12	3.258	0.778	2.214	4.302	2.520	3.996	2.655	3.861	2.736	3.780
13	3.336	0.770	2.303	4.369	2.606	4.066	2.740	3.932	2.819	3.853
14	3.407	0.762	2.385	4.429	2.684	4.130	2.817	3.997	2.896	3.918
15	3.472	0.755	2.459	4.485	2.756	4.188	2.887	4.057	2.966	3.978
16	3.532	0.749	2.527	4.537	2.821	4.243	2.952	4.112	3.030	4.034
17	3.588	0.743	2.591	4.585	2.883	4.293	3.012	4.164	3.090	4.086
18	3.640	0.738	2.650	4.630	2.940	4.340	3.068	4.212	3.145	4.135
19	3.689	0.733	2.706	4.672	2.994	4.384	3.121	4.257	3.197	4.181
20	3.735	0.729	2.757	4.713	3.043	4.427	3.170	4.300	3.246	4.224
21	3.778	0.724	2.807	4.749	3.091	4.465	3.217	4.339	3.292	4.264
22	3.819	0.720	2.853	4.785	3.136	4.502	3.261	4.377	3.336	4.302
23	3.858	0.716	2.897	4.819	3.179	4.537	3.303	4.413	3.378	4.338
24	3.895	0.712	2.940	4.850	3.220	4.570	3.343	4.447	3.417	4.373
25	3.931	0.709	2.980	4.882	3.258	4.604	3.382	4.480	3.455	4.407

Table 3: A coefficient of *MA-R* control limits for the unknown parameter σ when $i < w$

Sample Size (n)	d_2	d_3	$i = 1$		$i = 2$		$i = 3$		$i = 4$	
			D_9^*	D_{10}^*	D_9^*	D_{10}^*	D_9^*	D_{10}^*	D_9^*	D_{10}^*
2	1.128	0.853	0	3.269	0	2.604	0	2.310	0	2.134
3	1.693	0.888	0	2.574	0	2.113	0.092	1.908	0.213	1.787
4	2.059	0.880	0	2.282	0.093	1.907	0.260	1.740	0.359	1.641
5	2.326	0.864	0	2.114	0.212	1.788	0.357	1.643	0.443	1.557
6	2.534	0.848	0	2.004	0.290	1.710	0.420	1.580	0.498	1.502
7	2.704	0.833	0.076	1.924	0.347	1.653	0.466	1.534	0.538	1.462
8	2.847	0.820	0.136	1.864	0.389	1.611	0.501	1.499	0.568	1.432
9	2.970	0.808	0.184	1.816	0.423	1.577	0.529	1.471	0.592	1.408
10	3.078	0.797	0.223	1.777	0.451	1.549	0.552	1.448	0.612	1.388
11	3.173	0.787	0.256	1.744	0.474	1.526	0.570	1.430	0.628	1.372
12	3.258	0.778	0.284	1.716	0.493	1.507	0.586	1.414	0.642	1.358
13	3.336	0.770	0.308	1.692	0.510	1.490	0.600	1.400	0.654	1.346
14	3.407	0.762	0.329	1.671	0.526	1.474	0.613	1.387	0.665	1.335
15	3.472	0.755	0.348	1.652	0.539	1.461	0.623	1.377	0.674	1.326
16	3.532	0.749	0.364	1.636	0.550	1.450	0.633	1.367	0.682	1.318
17	3.588	0.743	0.379	1.621	0.561	1.439	0.641	1.359	0.689	1.311
18	3.640	0.738	0.392	1.608	0.570	1.430	0.649	1.351	0.696	1.304
19	3.689	0.733	0.404	1.596	0.578	1.422	0.656	1.344	0.702	1.298
20	3.735	0.729	0.414	1.586	0.586	1.414	0.662	1.338	0.707	1.293
21	3.778	0.724	0.425	1.575	0.593	1.407	0.668	1.332	0.713	1.287
22	3.819	0.720	0.434	1.566	0.600	1.400	0.673	1.327	0.717	1.283
23	3.858	0.716	0.443	1.557	0.606	1.394	0.679	1.321	0.722	1.278
24	3.895	0.712	0.452	1.548	0.612	1.388	0.683	1.317	0.726	1.274
25	3.931	0.709	0.459	1.541	0.617	1.383	0.688	1.312	0.729	1.271

Table 4: A coefficient of *MA-R* control limits for unknown parameter σ, s when $i \geq w$

Sample Size (n)	d_2	d_3	$w = 5$		$w = 10$		$w = 15$		$w = 20$	
			D_{11}^*	D_{12}^*	D_{11}^*	D_{12}^*	D_{11}^*	D_{12}^*	D_{11}^*	D_{12}^*
2	1.128	0.853	0	2.015	0.283	1.717	0.414	1.586	0.493	1.507
3	1.693	0.888	0.296	1.704	0.502	1.498	0.594	1.406	0.648	1.352
4	2.059	0.880	0.427	1.573	0.595	1.405	0.669	1.331	0.713	1.287
5	2.326	0.864	0.502	1.498	0.648	1.352	0.712	1.288	0.751	1.249
6	2.534	0.848	0.551	1.449	0.683	1.317	0.741	1.259	0.776	1.224
7	2.704	0.833	0.587	1.413	0.708	1.292	0.761	1.239	0.793	1.207
8	2.847	0.820	0.614	1.386	0.727	1.273	0.777	1.223	0.807	1.193
9	2.970	0.808	0.635	1.365	0.742	1.258	0.789	1.211	0.818	1.182
10	3.078	0.797	0.653	1.347	0.754	1.246	0.799	1.201	0.826	1.174
11	3.173	0.787	0.667	1.333	0.765	1.235	0.808	1.192	0.834	1.166
12	3.258	0.778	0.680	1.320	0.773	1.227	0.815	1.185	0.840	1.160
13	3.336	0.770	0.690	1.310	0.781	1.219	0.821	1.179	0.845	1.155
14	3.407	0.762	0.700	1.300	0.788	1.212	0.827	1.173	0.850	1.150
15	3.472	0.755	0.708	1.292	0.794	1.206	0.832	1.168	0.854	1.146
16	3.532	0.749	0.715	1.285	0.799	1.201	0.836	1.164	0.858	1.142
17	3.588	0.743	0.722	1.278	0.804	1.196	0.840	1.160	0.861	1.139
18	3.640	0.738	0.728	1.272	0.808	1.192	0.843	1.157	0.864	1.136
19	3.689	0.733	0.733	1.267	0.811	1.189	0.846	1.154	0.867	1.133
20	3.735	0.729	0.738	1.262	0.815	1.185	0.849	1.151	0.869	1.131
21	3.778	0.724	0.743	1.257	0.818	1.182	0.852	1.148	0.871	1.129
22	3.819	0.720	0.747	1.253	0.821	1.179	0.854	1.146	0.874	1.126
23	3.858	0.716	0.751	1.249	0.824	1.176	0.856	1.144	0.876	1.124
24	3.895	0.712	0.755	1.245	0.827	1.173	0.858	1.142	0.877	1.123
25	3.931	0.709	0.758	1.242	0.829	1.171	0.860	1.140	0.879	1.121

Table 5: Simulated data from an example of *R* and *MA-R* control charts

m	Simulated Data					R_j	MA-R	
							w = 2	*w* = 3
	1	2	3	4	5	for $i \geq 1$ UCL = 7.8551 LCL = 0	for $i \geq 2$ UCL = 6.6404 LCL = 0.7874	for $i \geq 3$ UCL = 6.1036 LCL = 1.3244
1	9.1363	8.9109	9.3844	11.4193	8.8520	2.5673	2.5673	2.5673
2	10.0774	10.0326	10.7481	10.2916	10.1049	0.7155	1.6414	1.6414
3	8.7859	10.5525	9.8076	10.1978	10.7223	1.9364	1.3260	1.7397
4	8.8865	11.1006	10.8886	11.5877	12.5855	3.6990	2.8177	2.1170
5	9.9932	11.5442	9.2352	9.1955	9.3331	2.3487	3.0239	2.6614
6	11.5326	10.0859	8.5977	10.6966	10.1873	2.9349	2.6418	2.9942
7	9.2303	8.5084	8.5776	10.8351	9.9175	2.3267	2.6308	2.5368
8	10.3714	9.2577	10.4882	9.7563	8.0670	2.4212	2.3740	2.5609
9	9.7744	8.9384	9.8226	10.2157	9.5610	1.2773	1.8493	2.0084
10	11.1174	12.3505	9.8039	8.8342	8.2053	4.1452	2.7113	2.6146
11	11.6808	5.7233	11.8160	9.2923	10.5408	6.0927	5.1190	3.8384
12	8.2239	8.3208	11.6504	8.3528	9.4760	3.4265	4.7596	4.5548
13	10.2002	12.7092	12.7579	6.8459	6.4996	6.2583	4.8424	5.2592
14	8.9109	7.8557	7.8836	11.0159	9.4287	3.1602	4.7093	4.2817
15	10.6070	11.9219	9.0628	10.5640	8.3373	3.5846	3.3724	4.3344
16	8.7993	10.2481	9.4551	10.0670	8.0416	2.2065	2.8956	2.9838
17	10.9799	12.8734	12.1968	7.3326	7.6872	5.5408	3.8737	3.7773
18	11.4787	6.0782	9.4443	12.2550	8.9329	6.1768	5.8588	4.6414
19	13.4238	9.6046	11.4031	10.7004	5.9947	7.4291	6.8030	6.3822
20	9.6118	7.5843	5.8964	9.4019	11.9285	6.0321	6.7306	6.5460
						$\overline{R} = 3.714$		

Table 5: Simulated data from an example of *R* and *MA-R* control charts (*Continued*)

m	MA-R				
	w = 4	*w* = 5	*w* = 10	*w* = 15	*w* = 20
	for $i \geq 4$ UCL = 5.7834 LCL = 1.6446	for $i \geq 5$ UCL = 5.5650 LCL = 1.8629	for $i \geq 10$ UCL = 5.0228 LCL = 2.4052	for $i \geq 15$ UCL = 4.7825 LCL = 2.6455	for $i \geq 20$ UCL = 4.6395 LCL = 2.7885
1	2.5673	2.5673	2.5673	2.5673	2.5673
2	1.6414	1.6414	1.6414	1.6414	1.6414
3	1.7397	1.7397	1.7397	1.7397	1.7397
4	2.2296	2.2296	2.2296	2.2296	2.2296
5	2.1749	2.2534	2.2534	2.2534	2.2534
6	2.7298	2.3269	2.3670	2.3670	2.3670
7	2.8273	2.6491	2.3612	2.3612	2.3612
8	2.5079	2.7461	2.3687	2.3687	2.3687
9	2.2400	2.2618	2.2474	2.2474	2.2474
10	2.5426	2.6211	2.4372	2.4372	2.4372
11	3.4841	3.2526	2.7898	2.7695	2.7695
12	3.7354	3.4726	3.0609	2.8243	2.8243
13	4.9807	4.2400	3.4931	3.0884	3.0884
14	4.7344	4.6166	3.4392	3.0936	3.0936
15	4.1074	4.5045	3.5628	3.1263	3.1263
16	3.8024	3.7272	3.4899	3.1022	2.4551
17	3.6230	4.1501	3.8113	3.4239	2.6037
18	4.3772	4.1338	4.1869	3.7066	2.8768
19	5.3383	4.9876	4.8021	3.9553	3.1514
20	6.2947	5.4771	4.9908	4.2009	3.2681

Table 6: Control limits of *R* and *MA-R* charts with *w* = 2, 3, 4, 5, 10, 15 and 20 for simulated data

***R* chart**	***MA-R* chart**		
	***w* = 2**	***w* = 3**	***w* = 4**
for *i* ≥ 1	**for *i* ≥ 2**	**for *i* ≥ 3**	**for *i* ≥ 4**
$UCL = D_4\overline{R}$ $= 2.115*3.714$ $= 7.8551$ $LCL = D_3\overline{R}$ $= 0*3.714$ $= 0$	$UCL = \left(1+3\frac{d_3}{d_2}\sqrt{\frac{1}{2}}\right)\overline{R}$ $= D_{10}^*\overline{R}$ $= 1.788*3.714$ $= 6\ 6406$ $LCL = \left(1-3\frac{d_3}{d_2}\sqrt{\frac{1}{2}}\right)\overline{R}$ $= D_9^*\overline{R}$ $= 0.212*3.714$ $= 0\ 7874$	$UCL = \left(1+3\frac{d_3}{d_2}\sqrt{\frac{1}{3}}\right)\overline{R}$ $= D_{10}^*\overline{R}$ $= 1.643*3.714$ $= 6.1021$ $LCL = \left(1-3\frac{d_3}{d_2}\sqrt{\frac{1}{3}}\right)\overline{R}$ $= D_9^*\overline{R}$ $= 0.357*3.714$ $= 1\ 3259$	$UCL = \left(1+3\frac{d_3}{d_2}\sqrt{\frac{1}{4}}\right)\overline{R}$ $= D_{10}^*\overline{R}$ $= 1.557*3.714$ $= 5\ 7827$ $LCL = \left(1-3\frac{d_3}{d_2}\sqrt{\frac{1}{4}}\right)\overline{R}$ $= D_9^*\overline{R}$ $= 0.443*3.714$ $= 1\ 6453$
***w* = 5**	***w* = 10**	***w* = 15**	***w* = 20**
for *i* ≥ 5	**for *i* ≥ 10**	**for *i* ≥ 15**	**for *i* ≥ 20**
$UCL = \left(1+3\frac{d_3}{d_2}\sqrt{\frac{1}{5}}\right)\overline{R}$ $= D_{12}^*\overline{R}$ $= 1.498*3.714$ $= 5.5636$ $LCL = \left(1-3\frac{d_3}{d_2}\sqrt{\frac{1}{5}}\right)\overline{R}$ $= D_{11}^*\overline{R}$ $= 0.502*3.714$ $= 1.8645$	$UCL = \left(1+3\frac{d_3}{d_2}\sqrt{\frac{1}{10}}\right)\overline{R}$ $= D_{12}^*\overline{R}$ $= 1.352*3.714$ $= 5.0214$ $LCL = \left(1-3\frac{d_3}{d_2}\sqrt{\frac{1}{10}}\right)\overline{R}$ $= D_{11}^*\overline{R}$ $= 0.648*3.714$ $= 2.4067$	$UCL = \left(1+3\frac{d_3}{d_2}\sqrt{\frac{1}{15}}\right)\overline{R}$ $= D_{12}^*\overline{R}$ $= 1.288*3.714$ $= 4.7837$ $LCL = \left(1-3\frac{d_3}{d_2}\sqrt{\frac{1}{15}}\right)\overline{R}$ $= D_{11}^*\overline{R}$ $= 0.712*3.714$ $= 2.6444$	$UCL = \left(1+3\frac{d_3}{d_2}\sqrt{\frac{1}{20}}\right)\overline{R}$ $= D_{12}^*\overline{R}$ $= 1.249*3.714$ $= 4.6388$ $LCL = \left(1-3\frac{d_3}{d_2}\sqrt{\frac{1}{20}}\right)\overline{R}$ $= D_{11}^*\overline{R}$ $= 0.751*3.714$ $= 2.7893$

Table 7: Flow width measurements data in the hard-bake process of *R* and *MA-R* control charts

m	**Simulated Data**					**R_j**	**MA-R**	
							***w* = 2**	***w* = 3**
	1	**2**	**3**	**4**	**5**	**for *i* ≥ 1** **UCL = 0.6367** **LCL = 0**	**for *i* ≥ 2** **UCL = 0.5383** **LCL = 0.0638**	**for *i* ≥ 3** **UCL = 0.4947** **LCL = 10.1073**
1	1.4483	1.5458	1.4538	1.4303	1.6206	0.1903	0.1903	0.1903
2	1.5435	1.6899	1.5830	1.3358	1.4187	0.3541	0.2722	0.2722
3	1.5175	1.3446	1.4723	1.6657	1.6661	0.3215	0.3378	0.2886
4	1.5454	1.0931	1.4072	1.5039	1.5264	0.4523	0.3869	0.3760
5	1.4418	1.5059	1.5124	1.4620	1.6263	0.1845	0.3184	0.3194
6	1.4301	1.2725	1.5945	1.5397	1.5252	0.3220	0.2533	0.3196
7	1.4981	1.4506	1.6174	1.5837	1.4962	0.1668	0.2444	0.2244
8	1.3009	1.5060	1.6231	1.5831	1.6454	0.3445	0.2557	0.2778
9	1.4132	1.4603	1.5808	1.7111	1.7313	0.3181	0.3313	0.2765
10	1.3817	1.3135	1.4953	1.4894	1.4596	0.1818	0.2500	0.2815
11	1.5765	1.7014	1.4026	1.2773	1.4541	0.4241	0.3030	0.3080
12	1.4936	1.4373	1.5139	1.4808	1.5293	0.0920	0.2581	0.2326
13	1.5729	1.6738	1.5048	1.5651	1.7473	0.2425	0.1673	0.2529
14	1.8089	1.5513	1.8250	1.4389	1.6558	0.3861	0.3143	0.2402
15	1.6236	1.5393	1.6738	1.8698	1.5036	0.3662	0.3762	0.3316
16	1.4120	1.7931	1.7345	1.6391	1.7791	0.3811	0.3737	0.3778
17	1.7372	1.5663	1.4910	1.7809	1.5504	0.2899	0.3355	0.3457
18	1.5971	1.7394	1.6832	1.6677	1.7974	0.2003	0.2451	0.2904
19	1.4295	1.6536	1.9134	1.7272	1.4370	0.4839	0.3421	0.3247
20	1.6217	1.8220	1.7915	1.6744	1.9404	0.3187	0.4013	0.3343
						$\overline{R} = 0.301$		

Table 7: Flow width measurements data in the hard-bake process of *R* and *MA-R* control charts (*Continued*)

m	MA-R				
	w **= 4**	***w*** **= 5**	***w*** **= 10**	***w*** **= 15**	***w*** **= 20**
	for *i* ≥ 4 **UCL = 0.4688** **LCL = 0.1333**	**for *i* ≥ 5** **UCL = 0.4511** **LCL = 0.1510**	**for *i* ≥ 10** **UCL = 0.4071** **LCL = 0.1950**	**for *i* ≥ 15** **UCL = 0.3876** **LCL = 0.2144**	**for *i* ≥ 20** **UCL = 0.3761** **LCL = 0.2260**
1	0.1903	0.1903	0.1903	0.1903	0.1903
2	0.2722	0.2722	0.2722	0.2722	0.2722
3	0.2886	0.2886	0.2886	0.2886	0.2886
4	0.3296	0.3296	0.3296	0.3296	0.3296
5	0.3281	0.3005	0.3005	0.3005	0.3005
6	0.3201	0.3269	0.3041	0.3041	0.3041
7	0.2814	0.2894	0.2845	0.2845	0.2845
8	0.2545	0.2940	0.2920	0.2920	0.2920
9	0.2879	0.2672	0.2949	0.2949	0.2949
10	0.2528	0.2666	0.2836	0.2836	0.2836
11	0.3171	0.2871	0.3070	0.2964	0.2964
12	0.2540	0.2721	0.2808	0.2793	0.2793
13	0.2351	0.2517	0.2729	0.2765	0.2765
14	0.2862	0.2653	0.2662	0.2843	0.2843
15	0.2717	0.3022	0.2844	0.2898	0.2898
16	0.3440	0.2936	0.2903	0.3025	0.2955
17	0.3558	0.3332	0.3026	0.2982	0.2952
18	0.3094	0.3247	0.2882	0.2901	0.2899
19	0.3388	0.3443	0.3048	0.2923	0.3001
20	0.3232	0.3348	0.3185	0.3012	0.3010

Table 8: Control limits of *R* and *MA-R* charts with *w* = 2, 3, 4, 5, 10, 15 and 20 for flow width measurements data in hard–bake process

R **chart**	***MA-R*** **chart**		
	w **= 2**	***w*** **= 3**	***w*** **= 4**
for *i* ≥ 1	**for *i* ≥ 2**	**for *i* ≥ 3**	**for *i* ≥ 4**
$UCL = D_4\overline{R}$ $= 2.115*0.301$ $= 0.6367$ $LCL = D_3\overline{R}$ $= 0*0.301$ $= 0$	$UCL = \left(1+3\frac{d_3}{d_2}\sqrt{\frac{1}{2}}\right)\overline{R}$ $= D_{10}^{*}\overline{R}$ $= 1.788*0.301$ = 0 5382 $LCL = \left(1-3\frac{d_3}{d_2}\sqrt{\frac{1}{2}}\right)\overline{R}$ $= D_{9}^{*}\overline{R}$ $= 0.212*0.301$ = 0 0639	$UCL = \left(1+3\frac{d_3}{d_2}\sqrt{\frac{1}{3}}\right)\overline{R}$ $= D_{10}^{*}\overline{R}$ $= 1.643*0.301$ = 0 4946 $LCL = \left(1-3\frac{d_3}{d_2}\sqrt{\frac{1}{3}}\right)\overline{R}$ $= D_{9}^{*}\overline{R}$ $= 0.357*0.301$ = 0 1075	$UCL = \left(1+3\frac{d_3}{d_2}\sqrt{\frac{1}{4}}\right)\overline{R}$ $= D_{10}^{*}\overline{R}$ $= 1.557*0.301$ = 0 4687 $LCL = \left(1-3\frac{d_3}{d_2}\sqrt{\frac{1}{4}}\right)\overline{R}$ $= D_{9}^{*}\overline{R}$ $= 0.443*0.301$ = 0 1334
w **= 5**	***w*** **= 10**	***w*** **= 15**	***w*** **= 20**
for *i* ≥ 5	**for *i* ≥ 10**	**for *i* ≥ 15**	**for *i* ≥ 20**
$UCL = \left(1+3\frac{d_3}{d_2}\sqrt{\frac{1}{5}}\right)\overline{R}$ $= D_{12}^{*}\overline{R}$ $= 1.498*0.301$ $= 0.4509$ $LCL = \left(1-3\frac{d_3}{d_2}\sqrt{\frac{1}{5}}\right)\overline{R}$ $= D_{11}^{*}\overline{R}$ $= 0.502*0.301$ $= 0.1510$	$UCL = \left(1+3\frac{d_3}{d_2}\sqrt{\frac{1}{10}}\right)\overline{R}$ $= D_{12}^{*}\overline{R}$ $= 1.352*0.301$ $= 0.4070$ $LCL = \left(1-3\frac{d_3}{d_2}\sqrt{\frac{1}{10}}\right)\overline{R}$ $= D_{11}^{*}\overline{R}$ $= 0.648*0.301$ $= 0.1951$	$UCL = \left(1+3\frac{d_3}{d_2}\sqrt{\frac{1}{15}}\right)\overline{R}$ $= D_{12}^{*}\overline{R}$ $= 1.288*0.301$ $= 0.3877$ $LCL = \left(1-3\frac{d_3}{d_2}\sqrt{\frac{1}{15}}\right)\overline{R}$ $= D_{11}^{*}\overline{R}$ $= 0.712*0.301$ $= 0.2144$	$UCL = \left(1+3\frac{d_3}{d_2}\sqrt{\frac{1}{15}}\right)\overline{R}$ $= D_{12}^{*}\overline{R}$ $= 1.249*0.301$ $= 0.3760$ $LCL = \left(1-3\frac{d_3}{d_2}\sqrt{\frac{1}{15}}\right)\overline{R}$ $= D_{11}^{*}\overline{R}$ $= 0.751*0.301$ $= 0.2261$

4 Discussion and Conclusions

In this research, we propose a modified chart based on a moving average chart (*MA*) with the range for detecting a change in process variability by combining the features of *MA* and *R* charts, namely the *MA-R* chart. The prompt tables of coefficient for the *MA-R* charts are supplied for both cases of known and unknown parameter σ, s with different values of sample size (n) and width (w). The performance comparison of the *MA-R* chart versus with *R* chart, *S* chart, and *MA-S* chart by using two applications; simulated data and flow width measurement data. The comparison shows that the proposed chart is superior to the *R* and *S* charts. Also, the *MA-R* chart performs better for small as well as large sample sizes for both small and moderate shifts in process variability. Therefore, the *MA-R* chart is an effective alternative to *R* and other charts due to the simpler calculation and interpretation. However, the *S* and *MA-S* charts are very informative when compared with *R* and *MA-R* charts for large subgroups. Conversely, the *MA-R* is proper for small subgroups due to its easy calculation and explanation as well as it helps to improve the detection performance with varying the width (w). In further research, the most popular criterion to compare the performance of control charts is Average run length (ARL) will be shown as an explicit formula of ARL and design new control charts for better performance to detect a change in process variability such a Double Moving Average – Range control chart.

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C.C.: data curation, data analysis, validation, investigation, methodology, writing; S.S.: conceptualization, investigation, reviewing and editing; writing an original draft, reviewing and editing, funding acquisition, Y.A.: research design, conceptualization, and project administration. All authors have read and agreed to the published version of the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

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