Research Article

Explicit Formulas of Moving Average Control Chart for Zero Modified Geometric Integer Valued Autoregressive Process

Saowanit Sukparungsee
Department of Applied Statistics, Faculty of Applied Science, King Mongkut's University of Technology North Bangkok, Bangkok, Thailand

Suganya Phantu*
Faculty of Science Energy and Environmental, King Mongkut's University of Technology North Bangkok, Rayong, Thailand

* Corresponding author. E-mail: suganya.p@sciee.kmutnb.ac.th DOI: 10.14416/j.asep.2023.09.004
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Abstract
This research presents precise formulas to calculate the average time to signal (ATS) of the moving average control chart (MA chart) for detecting changes in the autocorrelation of count data when the process has zero inflation and zero deflation. Thus, a zero-modified geometric integer value autoregressive order 1 (ZMGINAR (1)) process is a suitable geometrical alternative for autocorrelated count data with an enormous (or shortfall) number of zeros. The average time to signal is a traditional control chart performance; the mean of the observations taken before a process signal that it is beyond the control limit. The numerical results demonstrate the effectiveness of the control limit in detecting changes in the effect of inflation and deflation of zeros. The usefulness of a control chart in detecting variations in the model of the process can be illustrated by the actual data sample of count data.

Keywords: Average time to signal, INAR(1) process, Integer-valued time series, MA chart, Zero-modified geometric distribution

1 Introduction

In statistical process control (SPC), there are many sample data in the form of count data. Count time series consist of zero inflation and zero deflation, and such states can occur in economics, industrial, business, and other data fields. Over time, there have been many studies to develop models suitable for zero inflation and friction data. The most suitable model for this data type is the geometric INAR model with zero modification geometry as the increment distribution called the ZMGINAR(1) model proposed by Berreto-Souza [1].

In the quality control process, there may be a change in the mean or variance of the data. The most popular statistical tool to detect changes is the control chart, which is used to detect changes in data parameters. For instance, the *Shewhart chart* [2] can effectively detect large process variations as it considers variations in current data such as *c*, *u*, *p*, and *np charts*. The *c chart* monitors the number of defects in a sample, while the *u chart* monitors the average number per sample unit. The *c chart* is similar to the np chart except that it counts defects instead of defectives. A p chart monitors the proportion of defectives in a lot or batch and counts the number of non-conforming units in a lot or batch. The np chart monitors the number of defects. However, both should look the same for the same data set with constant sample sizes. Subsequently, a control chart that detects small process changes was developed, namely the moving average (MA) control [3]. The usage condition is to consider different moving averages (k) that can detect small changes.

The efficiency of the control chart can be determined from the average time to signal (ATS), divided into two categories: when the process is in control and when it is out of control [4]. When the process is in control, the above values should be high in normal circumstances, and on the contrary, this value should be minimal when the process is out of control. In the past, there have been many analytical methods for calculating ATS. The most widely used and accurate method is Monte Carlo (MC) simulation. However, this method has limitations in using large amounts of observations and time.

Furthermore, many methods are presented. The Markov chain approach (MCA) is a popular and effective technique applying the definition of matrix inversion to the Markov chain principles. There is no theoretical effect on accuracy, but the results are compared with MC [5]. In addition, the integral equation (IE) is the modern method using basic mathematical formulas and the central limit theorem, which is another method that can accurately measure the performance of a control chart [6]. The methods described above are suitable for determining the efficiency of control charts, but they are not suitable for all control charts in optimizing as the process changes.

Considerable research has examined the effectiveness of the control chart for autocorrelation detection when the process has zero-increasing or decreasing counting time series. Rakitzis *et al.*, [7]–[9] studied the efficiency of the CUSUM chart for one-sided and two-sided cases using the MC method to determine the ATS of zeros processes. Saowanit [10] proposed determining the average run length of the CUSUM chart by the MCA method based on Poisson counting data with inflated zero values. Phanyam [11] analyzed a formula to determine the EWMA chart's performance with the IE method for the autocorrelated processes. Areepong and Sukparungsee [12] proposed a method for calculating the ATS using the IE principle of the EWMA chart and compared the efficiency of the obtained results with the MC methods. In addition, Chananet *et al.*, [13] use MC to detect process variation.

Additionally, many studies have been conducted on calculating control chart performance measures to obtain accurate numerical results. Areepong and Sukparungsee [14] analyzed the average run length formula of the MA chart to be used for the process with inflated zero values. Efficiency analysis of the MA chart found that as the zero change process increases, the length of the MA chart used for detecting the change decreases. Chananet *et al.*, [15] studied the estimation formula for the performance measure of MA charts with zero data of the negative binomial distribution. The resulting formula is easy to apply to data with zero. Phantu *et al.*, [16] studied explicit expressions to calculate the average run length values for the MA chart using Poisson integer-valued autoregressive model. To make it more convenient to find the efficiency of the MA chart. Sukparungsee *et al.*, [17] propose an exact formula to determine the average run length of the MA chart for an integer-valued moving average of order 1 (PoiINMA(1)) and compare the accuracy with the MC method. As a result, the generated formula is as accurate as the MC method but takes less time to process. Areepong [18] finds a formula for calculating the performance measure of the MA chart to determine the process mean of the first-order integer-valued autoregressive with a zero-inflated Poisson model (ZIPNAR(1)). Raweesawat and Sukparungsee [19] proposed an accurate formula to estimate the performance of the DMA chart by using a double-moving average of the hyper zero models. So it can be used more efficiently. Wiwek *et al.*, [20] proposed applying exact formulas of average time to out-of-control signals for a compound control chart. The research studies above have shown that determining the efficiency of control chat using formulas is accurate and consistent with the MC method. It is also a popular method for moving average control charts because it is convenient, fast, and easy to use.

The purpose of this paper is to study the precise formula of the MA chart when the data is observed as a zero-modified geometric integer value autoregressive order 1 (ZMGINAR (1)) model and to compare the performance of the MA chart with the *Shewhart chart* to detect changes in the mean parameter of the process with inflation or deflation of zero. The paper is organized into the section as follows. The first section is the introduction. The second section describes the zero-modified geometric integer value autoregressive order 1 (ZMGINAR (1)) model, while the third section presents the control chart for ZMGINAR (1) model. Next, the fourth section analysis the precise formula of the MA chart. Later, section fifth illustrates the results of the performance of the control chart and the optimization of the parameters of the MA chart, including using a control chart of data with inflation and deflation of zeros. Finally, the conclusion of this research is provided in part six.

2 The first order Zero-Modified Geometric First-Order Integer Valued Autoregressive Process

This section introduces a zero-modified geometric integer-valued autoregressive of order 1, denominate ZMGINAR(1), the pioneers presented by Berreto-Souza [1]. Let $\{C_t\}_{t\geq 0}$ is a stationary of a random variable with ZMG(α, β); see detailed McKenzie [21], Al-Osh and Alzaid [22], Bourguinon and Weiβ [23], and Rakitzis *et al.* [24]. By definition, the ZMGINAR(1) process can be defined as

$$c_t = \theta * c_{t-1} + \omega_t,$$

For $t \geq 0$, $\{\omega_t\}_{t\geq 0}$ is identically independent random variables of the count series; C_{t-1} and C_t independent for all $t \geq 1$. The mean and variance are

$$E(C) = \mu = \alpha(1-\beta)$$

and

$$Var(C) = \sigma^2 = \alpha(1-\beta)\left[1+\alpha(1+\beta)\right].$$

Where α is the probability of zero modified geometric distribution, and β is the probability of zero. Thus, if $\alpha \in (0,1)$ and $\beta \in (-1/\alpha, 0)$, ZMGINAR(1) model presents underdispersion, if $\alpha \in (0,1)$ and $\beta = -1$ ZMGINAR(1) model is equidispersed, and for $\beta > 0$ and $\alpha \in (-1,1)$ ZMGINAR(1) model presents overdispersion. The approximation of the parameters of the ZMGINAR(1) model can calculate by following as

$$\alpha = \frac{(n-1)\sum_{t=2}^{n} C_t C_{t-1} - \sum_{t=2}^{n} C_t \sum_{t=2}^{n} C_{t-1}}{(n-1)\sum_{t=2}^{n} C_{t-1}^2 - \left(\sum_{t=2}^{n} C_{t-1}\right)^2} \quad (1)$$

and

$$\beta = 1 - \frac{\sum_{t=2}^{n} C_t - \alpha \sum_{t=2}^{n} C_{t-1}}{\mu(n-1)(1-\alpha)}. \quad (2)$$

3 The Control Chart for ZMGINAR (1) Model

The control charts used to measure the shift magnitude of the ZMGINAR(1) process are:

3.1 *Shewhart control chart*

The Shewhart control chart is proposed by Shewhart [2]. It is suitable for a measure or quantitative data. Let C is a sample of the ZMGINAR(1) process. The statistics of the Shewhart chart of the ZMGINAR(1) model is

$$Shewhart_i = C_1, C_2, ..., C_n.$$

The expectations and variance of the Shewhart chart can be calculated as follows:

$$E(Shewhart) = \alpha(1-\beta)$$

and

$$Var(Shewhart) = \frac{\alpha(1-\beta)\left(1+\alpha(1+\beta)\right)}{n}.$$

Thus, the upper and lower bound of the Shewhart chart for the ZMGINAR(1) process can be represented as follows:

$$UCL/LCL = \alpha(1-\beta) \pm H_1\sqrt{\alpha(1-\beta)\left[1+\alpha(1+\beta)\right]}$$

where $\alpha(1-\beta)$ is the mean of the ZMGINAR(1) model. H_1 is the coefficient of the control limits of the Shewhart chart.

3.2 *Moving average control chart*

The moving average (MA) control chart uses a moving average, where the previous sample value is averaged with the current data, to create a smooth stand-in state for the current process variable, which smoothes the data to reduce process impact [3], [4]. Suppose C is individual observations from the ZMGINAR (1) model. The value of the moving average at the time is defined as

$$MA_i = \begin{cases} \frac{1}{i}\sum_{j=1}^{i} C_j & ; \quad i < k \\ \frac{1}{k}\sum_{j=i-k+1}^{i} C_j & ; \quad i \geq k \end{cases}, \quad (3)$$

where k is the length of the moving average. When the process is in-control, the expectation and variance of the MA chart are

$$E(MA_i) = \alpha(1-\beta)$$

and

$$Var(MA_i) = \begin{cases} \dfrac{\alpha(1-\beta)\left(1+\alpha(1+\beta)\right)}{i}; & i < k \\ \dfrac{\alpha(1-\beta)\left(1+\alpha(1+\beta)\right)}{k}; & i \geq k \end{cases}.$$

The monitoring statistics of upper and lower control limits of the MA chart are

$$UCL/LCL = \begin{cases} \alpha(1-\beta) \pm H_2\sqrt{\dfrac{\alpha(1-\beta)\left[1+\alpha(1+\beta)\right]}{i}}; i < k \\ \alpha(1-\beta) \pm H_2\sqrt{\dfrac{\alpha(1-\beta)\left[1+\alpha(1+\beta)\right]}{k}}; i \geq k \end{cases} \quad (4)$$

where H_2 is a coefficient of control limit determined based on the desired in-control ATS (ATS_0), the MA chart can detect process change when the signal is out of control, divided into 2 cases; when $MA_i < LCL$ or $MA_i > UCL$.

4 A Precise Formula for ATS of MA chart for ZMGINAR(1) Model

This section analysis a method for calculating the ATS from the precise formula of the ZMGINAR(1) model. The precise formula consists of two cases; in-control ATS (ATS_0) and out-of-control ATS (ATS_1) processes. The central limit theorem (CLT) can analyze the formulation of ATS, and given OC is out of control, limit. The limitation of CLT is that it assumes that the sample is independent. More details can be found by Khoo [5], Areepong and Sukparungsee [14], Chananet *et al.* [15], Phantu *et al.* [16], Sukparungsee *et al.* [17], Areepong [18], Raweesawat and Sukparungsee [19] and Wiwek *et al.* [20].

Let ATS = A, then the probability of the sample going outside the control limit is $1/A$. Note that the overall ATS of the MA chart has the following definition:

$$\frac{1}{A} = \frac{1}{A^+} + \frac{1}{A^-} \quad (5)$$

where A^+ and A^- are the ATS of the upper and lower sides of the MA schemes. From Equation (5), the OC of the region is divided into 2 cases for several A of MA charts. Thus, the ATS of the ZMGINAR(1) model can be obtained as

$$\frac{1}{A} = \frac{1}{A}P(OC\ \ limit\, at\, time\ i < k) + \left[\frac{n-(k-1)}{A}\right]P(OC\, limit\, at\, time\, i \geq k). \quad (6)$$

From Equation (6), the monitoring statistics of the MA chart for the OC state can be replaced by Equation (3) in Equation (6). Therefore, the probability of statistics for the MA chart is

$$\frac{1}{A} = \frac{1}{A}\left\{\sum_{i=1}^{k-1}\left[P\left(\frac{\sum_{j=1}^{i} C_j}{i} > UCL_i\right) + P\left(\frac{\sum_{j=1}^{i} C_j}{i} < LCL_i\right)\right]\right\} + \left[\frac{A-(2w-1)}{A}\right]\left\{P\left(\frac{\sum_{j=i-k+1}^{i} C_j}{k} > UCL_k\right) + P\left(\frac{\sum_{j=i-k+1}^{i} C_j}{k} > UCL_k\right)\right\}. \quad (7)$$

Again by the properties of the control limit of the MA chart, it can be substituted from Equation (4) into Equation (7) and then rewritten as follows:

$$= \frac{1}{A}\left\{\sum_{i=1}^{k-1}\left[P\left(\frac{\sum_{j=1}^{i} C_j}{i} > \alpha(1-\beta) + H_2\sqrt{\frac{\alpha(1-\beta)\left[1+\alpha(1+\beta)\right]}{i}}\right) + P\left(\frac{\sum_{j=1}^{i} C_j}{i} < \alpha(1-\beta) - H_2\sqrt{\frac{\alpha(1-\beta)\left[1+\alpha(1+\beta)\right]}{i}}\right)\right]\right\}$$
$$+ \left[\frac{A-(k-1)}{A}\right]\left\{P\left(\frac{\sum_{j=i-k+1}^{i} C_j}{k} > \alpha(1-\beta) + H_2\sqrt{\frac{\alpha(1-\beta)\left[1+\alpha(1+\beta)\right]}{k}}\right) + P\left(\frac{\sum_{j=i-k+1}^{i} C_j}{k} < \alpha(1-\beta) - H_2\sqrt{\frac{\alpha(1-\beta)\left[1+\alpha(1+\beta)\right]}{k}}\right)\right\}.$$

By definition, The CLT can be applied to derive the probability of OC for the MA chart. Then, the precise formulas of the MA chart for the ZMGINAR(1) model can be computed by

$$\frac{1}{A}=\frac{1}{A}\left\{\sum_{i=1}^{k-1}\left[P\left(Z_1>\frac{UCL_{i<k}-\alpha(1-\beta)}{\sqrt{\frac{\alpha(1-\beta)\left[1+\alpha(1+\beta)\right]}{i}}}\right)+P\left(Z_1<\frac{LCL_{i<k}-\alpha(1-\beta)}{\sqrt{\frac{\alpha(1-\beta)\left[1+\phi(1+\beta)\right]}{i}}}\right)\right]\right\}+\left[\frac{A-(k-1)}{A}\right]\left\{P\left(Z_2>\frac{UCL_{i\geq k}-\alpha(1-\beta)}{\sqrt{\frac{\alpha(1-\beta)\left[1+\alpha(1+\beta)\right]}{k}}}\right)+P\left(Z_2<\frac{LCL_{i\geq k}-\alpha(1-\beta)}{\sqrt{\frac{\alpha(1-\beta)\left[1+\alpha(1+\beta)\right]}{k}}}\right)\right\}. \quad (8)$$

According to Equation (8), let

$$G=\sum_{i=1}^{k}\left[P\left(Z_1>\frac{UCL_{i<k}-\alpha(1-\beta)}{\sqrt{\frac{\alpha(1-\beta)\left[1+\alpha(1+\beta)\right]}{i}}}\right)+P\left(Z_1<\frac{LCL_{i<k}-\alpha(1-\beta)}{\sqrt{\frac{\alpha(1-\beta)\left[1+\alpha(1+\beta)\right]}{i}}}\right)\right] \quad (9)$$

and

$$T=P\left(Z_2>\frac{UCL_{i\geq k}-\alpha(1-\beta)}{\sqrt{\frac{\alpha(1-\beta)\left[1+\alpha(1+\beta)\right]}{k}}}\right)+P\left(Z_2<\frac{LCL_{i\geq k}-\alpha(1-\beta)}{\sqrt{\frac{\alpha(1-\beta)\left[1+\alpha(1+\beta)\right]}{k}}}\right). \quad (10)$$

In order to calculate the ATS for MA chart precise formulas, it is helpful to replace G from Equation (9) and T from Equation (10) in Equation (6). Therefore, it can be written as

$$\frac{1}{A}=\frac{1}{A}G+\frac{A-(k-1)}{A}T$$

$$A=\frac{(1-G)}{T}+(k+1).$$

Given that ATS = A, Then

$$ATS=(1-G)T^{-1}+(k+1),\ \ k\neq\frac{A}{2}-1. \quad (11)$$

Finally, the precise formula of the MA chart for the ZMGINAR(1) model can be evaluated by formula Equation (11)

$$ATS=\left[1-\sum_{i=1}^{k-1}\left[P\left(Z_1>\frac{UCL_{i<k}-\alpha\left(1-\beta\right)}{\sqrt{\frac{\alpha\left(1-\beta\right)\left[1+\alpha\left(1+\beta\right)\right]}{i}}}\right)+P\left(Z_1<\frac{LCL_{i<k}-\alpha\left(1-\beta\right)}{\sqrt{\frac{\alpha\left(1-\beta\right)\left[1+\beta\left(1+\beta\right)\right]}{i}}}\right)\right]\right]\times\left[P\left(Z_2>\frac{UCL_{i\geq k}-\alpha\left(1-\beta\right)}{\sqrt{\frac{\alpha\left(1-\beta\right)\left[1+\alpha\left(1+\beta\right)\right]}{k}}}\right)+P\left(Z_2<\frac{LCL_{i\geq k}-\alpha\left(1-\beta\right)}{\sqrt{\frac{\alpha\left(1-\beta\right)\left[1+\alpha\left(1+\beta\right)\right]}{k}}}\right)\right]^{-1}+\left(k-1\right).$$

Let $\mu=\alpha(1-\beta)$, when the situation is in-control state, then ATS = ATS0, replace the value of parameter μ with μ_0 The processed explicit formulas of ATS_0 of the MA chart for the ZMGINAR(1) model is computed by Equation (12).

$$ATS_0 = \left[1 - \sum_{i=1}^{k-1}\left[P\left(Z_1 > \frac{UCL_{i<k} - \mu_0}{\sqrt{\frac{\alpha(1-\beta)[1+\alpha(1+\beta)]}{i}}}\right) + P\left(Z_1 < \frac{LCL_{i<k} - \mu_0}{\sqrt{\frac{\alpha(1-\beta)[1+\beta(1+\beta)]}{i}}}\right)\right]\right] \times \left[P\left(Z_2 > \frac{UCL_{i\geq k} - \mu_0}{\sqrt{\frac{\alpha(1-\beta)[1+\alpha(1+\beta)]}{k}}}\right) + P\left(Z_2 < \frac{LCL_{i\geq k} - \mu_0}{\sqrt{\frac{\alpha(1-\beta)[1+\alpha(1+\beta)]}{k}}}\right)\right]^{-1} + (k-1). \quad (12)$$

When the situation is an out-of-control state, then ATS = ATS_1, replace the value of parameter μ with μ_1. The shift of parameter of ZMGINAR(1) process in this research is $\mu_1 = \mu_0 + \delta$, with $\delta = \{0.02, 0.04, 0.06, 0.08, 0.1, 0.5, 1.0, 1.5, 2.0\}$. The explicit formula of the ATS1 of the MA chart is

$$ATS_1 = \left[1 - \sum_{i=1}^{k-1}\left[P\left(Z_1 > \frac{UCL_{i<k} - \mu_1}{\sqrt{\frac{\alpha(1-\beta)[1+\alpha(1+\beta)]}{i}}}\right) + P\left(Z_1 < \frac{LCL_{i<k} - \mu_1}{\sqrt{\frac{\alpha(1-\beta)[1+\alpha(1+\beta)]}{i}}}\right)\right]\right] \times \left[P\left(Z_2 > \frac{UCL_{i\geq k} - \mu_1}{\sqrt{\frac{\alpha(1-\beta)[1+\alpha(1+\beta)]}{k}}}\right) + P\left(Z_1 < \frac{LCL_{i<k} - \mu_1}{\sqrt{\frac{\alpha(1-\beta)[1+\alpha(1+\beta)]}{k}}}\right)\right]^{-1} + (k-1). \quad (13)$$

5 The Numerical Results

5.1 *Average time to signal of MA chart for ZMGINAR(1) model.*

In this part, the performance of the MA chart for the ZMGINAR(1) model is calculated by the precise formula in Equations (12) and (13), respectively, using the Mathematical software® [25]. Monte Carlo Simulation calculated the numerical result of the Shewhart chart. As previously described, attention depends on detecting a shift in parameters μ from ZMGINAR(1) model. It is assumed that there are no shifts in the parameter β, where $\beta > 0$ are the process of an excessive number of zeros and $\beta < 0$ are the process of a deficit number of zeros. In this research, consider the following values of β, –0.05 and 0.05. The shift of magnitudes in the study is $\mu_1 = \mu_0 + \delta$, with $\delta = \{0.02, 0.04, 0.06, 0.08, 0.1, 0.5, 1.0, 1.5, 2.0\}$. Table 1 summarizes the numerical results for the following configurations: $\alpha = 0.3, \beta = -0.05, 0.05$. Table 2 contains the numerical results for the following configurations: $\alpha = 0.5$, $\beta = -0.05, 0.05$. The performance comparison results from the above table can be summarized as follows: the MA chart detects average processing shifts more effectively than the Shewhart chart at all transition levels. For the ZMGINAR(1) model, the change is moderate when the process change exceeds 1. As a result, the MA chart detects changes better than the Shewhart chart.

Table 1: Comparison of average time to signal of control chart of ZMGINAR(1) model for $\alpha = 0.3$, $ATS_0 = 370$ and $k = 3$

δ	β			
	–0.05		**0.05**	
	Shewhart	**MA**	**Shewhart**	**MA**
0.00	370.400	370.398	370.400	370.398
0.02	332.785	*250.811*	344.183	*249.429*
0.04	305.931	*174.086*	310.557	*172.981*
0.06	286.143	*124.097*	288.743	*123.550*
0.08	241.326	*90.878*	243.298	*90.792*
0.10	219.528	*68.309*	221.526	*68.529*
0.50	146.286	*5.088*	147.342	*5.296*
1.00	110.342	*2.430*	112.349	*2.505*
1.50	75.941	*1.860*	77.625	*1.902*
2.00	32.183	*1.617*	34.956	*1.647*

Note: Italic is a minimum of ATS_1.

Table 2: Comparison of average time to signal of control chart of ZMGINAR(1) model for $\beta = 0.5$, $ATS_0 = 370$ and $k = 3$

δ	β			
	–0.05		0.05	
	Shewhart	MA	Shewhart	MA
0.00	370.400	370.398	370.400	370.398
0.02	325.467	*286.258*	327.462	*284.925*
0.04	306.172	*222.780*	307.948	*221.368*
0.06	276.334	*174.913*	277.926	*173.885*
0.08	239.165	*138.725*	340.714	*138.178*
0.10	206.742	*43.510*	208.363	*111.110*
0.50	152.469	*6.560*	153.948	*9.240*
1.00	107.315	*3.252*	109.765	*3.500*
1.50	79.210	*2.505*	80.255	*2.505*
2.00	33.208	*1.902*	35.971	*1.902*

Note: Italic is a minimum of ATS_1.

5.2 *Optimal design of MA chart for ZMGINAR(1) model*

The optimal parameter of the MA chart can be calculated from the precise formula in Equation (12). To illustrate the optimization of the ZMGINAR(1) model parameter in case of zero inflation and zero deflation. In this research, the following values of β are considered: $\beta = \{-0.1, 0.1\}$. The shift of magnitude in the research is $\mu_1 = \mu_0 + \delta$, with $\delta = \{0.02, 0.04, 0.06, 0.08, 0.1, 0.5, 1.0, 1.5, 2.0\}$. Table 3 contains the optimal results of the MA chart following configurations: $\alpha = 0.3$, $\beta = -0.1$, and $ATS_0 = 370$. Table 4 contains the optimal results of the MA chart following configurations: $\alpha = 0.3$, $\beta = 0.1$, and $ATS_0 = 370$. The results of finding the optimal parameter for the length of the moving average (k) of the MA chart found that the moving average (k) decreased when the change level increased because it resulted in the minimal ATS_1 value. In contrast, the moving average (k) length increased when the change level decreased.

5.3 *Empirical illustrations*

This section presents the utilization of the application for the ZMGINAR(1) model. The time series of actual data are observed. The sample data are obtained from the crime data part of the forecasting principles site (www.forecastingprinciples.com)—the total number of raids reported by the 34th police vehicle battery in Pittsburgh. The data section consisted of 144 samples, starting from January 1990 through December 2001, shown in Figure 1.

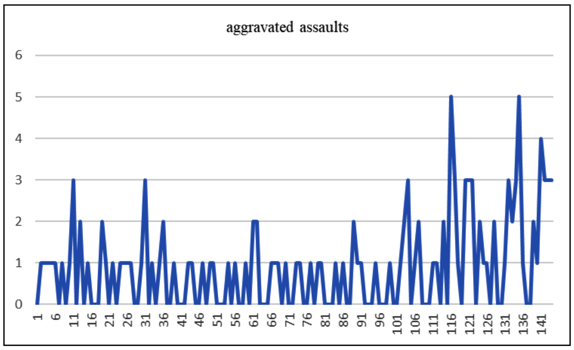

Figure 1: Count of aggregated assaults data series.

Table 3: Optimal design parameter and ATS_1 of MA chart of ZMGINAR(1) model for $\alpha = 0.3$, $\beta = -0.1$ and $ATS_0 = 370$

δ	k	H	ATS_1
0.02	24	3	243.182
0.04	22	3	187.340
0.06	21	3	135.486
0.08	19	3	109.431
0.10	13	3	74.914
0.50	5	3	10.842
1.00	3	3	3.415
1.50	3	3	2.149
2.00	2	3	1.847

Table 4: Optimal design parameter and ATS1 of MA chart of ZMGINAR(1) model for $\alpha = 0.3$, $\beta = 0.1$ and $ATS_0 = 370$

δ	k	H	ATS_1
0.02	24	3	271.421
0.04	20	3	223.483
0.06	18	3	187.346
0.08	12	3	124.617
0.10	6	3	93.746
0.50	4	3	14.966
1.00	3	3	5.218
1.50	3	3	2.412
2.00	2	3	1.543

The coefficient parameter of the ZMGINAR(1) model is calculated in Equations (1) and (2). The estimates of the parameters of the monthly number of the 34th police cart beat in Pittsburgh are $\alpha = 0.227$ and $\beta = -0.469$. Determine the coefficient of the Shewhart

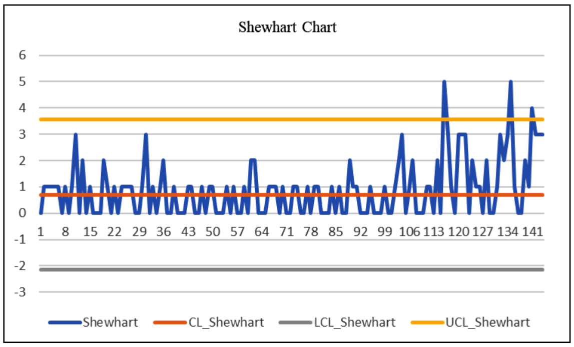

Figure 2: The performance of the Shewhart chart.

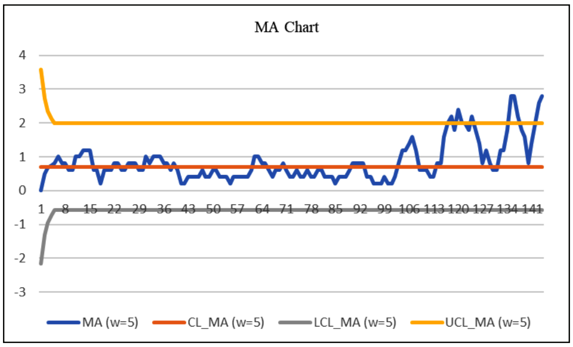

Figure 3: The performance of the MA chart.

and MA charts in detecting data changes equal to 3. The LCL and UCL of the Shewhart chart are –2.15 and 3.57, respectively. Assume that the LCL and UCL of MA charts have a moving average of 5. The performance of the Shewhart and MA chart is shown in Figures 2 and 3, respectively. The results show that the first sample outside of the control limit of the Shewhart and MA chart is sample no. 117. MA charts are suitable for detecting small to moderate changes, and MA charts are also easy to calculate chart statistics and control chart boundaries. The method used to measure the efficiency of the chart from the explicit formula is convenient and fast to use as well.

6 Conclusions

This paper proposed the precise formula to calculate the ATS of the MA chart for the ZMGINAR(1) model with inflated and deflated zeros. Comparing the control chart for ZMGINAR(1) model found that the MA chart is more effective at detecting process change than the Shewhart chart for all cases. In addition, the precise formula can help conveniently calculate the appropriate parameter (k) of the MA chart when the process changes. Implementing the Shewhart and MA charts to measure the performance of data change detection showed that the MA and Shewhart charts are equally effective. Future research can use count series data (e.g., financial and economic) and control charts to detect small process changes. The presented formula can be applied to other control charts and numerical data.

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Author Contributions

S.S.: conceptualization of research topic, research design, investigation, reviewing and editing; S.P.: methodology, data analysis, data curation, writing and revising contents, funding acquisition, project administration. All authors have read and agreed to the published version of the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

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