Research Article

Explicit Formulas of Average Run Length for Triple Moving Average Control Chart to Monitor Changes in Mean Parameter of Normal and Non-normal Process

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Abstract
In this research, a formula is presented for calculating the average run length of a triple moving average (TMA) control chart, which is used to measure control chart performance and determine control chart width. To apply appropriately to data in various processes, a TMA control chart is used to detect small changes in a process by finding the average value of the data over a specified period of time (w). However, determining measure control chart performance and control chart width is difficult because the formula for calculation is not created. The performance of the control charts is measured using the standard normal distribution probability properties and compared with the moving average (MA) control charts and the double moving average (DMA) control charts. The analysis results show that TMA is very effective at detecting small changes, when w is set to have a small value. The study determines that the data within the process are normally and non-normally distributed. Detecting changes on real data by TMA charts has the best detection performance compared to MA and DMA.

Keywords: Average run length, Efficiency, Detection, Mean, Time-varying chart

1 Introduction

In the midst of traveling to a future world filled with disruptive technologies, along with the post-COVID-19 situation, industrial and service business sectors in various areas around the world have set goals to compete for market share. The solution to this crisis is the "quality" of products and services, therefore, developing the industry is critical to meet all needs. Monitoring the changing times and emergencies, including industry guidelines will meet the production needs of the future. It must be aware to achieve the same standard of quality throughout the production process. This makes consumers trustworthy and accepts the product as a by-product of the evolution of various technologies that occur in the Industry 4.0 era, in terms of machines, equipment, and all information. They can be connected through the IoT network, sending product-related information through the production chain. It uses various types of robots to produce products according to specific customer needs. Machinery that can quickly adjust production as needed, including collecting production status information from different connections and using those data for analysis for remote monitoring, reducing maintenance costs and increasing efficiency in monitoring the production process.

Therefore, in controlling the quality process of the production process in Industry 4.0 plants, enterprises must be extremely aware of the importance of product quality and service especially those involved in the business establishment. The importance of using statistical quality control tools (Statistical Quality Control: SQC) must be considered to control, monitor, detect, and improve the quality of the process. The most effective statistical tools are control charts and sample selection plans for product quality inspection. In general, control charts can be divided into two types according to the quality characteristics that are detected. A variable control chart is a chart used to control the production process when the measured quality properties or characteristics can be weighed, such as the diameter of the workpiece, average volume of drinking water, average volume of packaging the average, lifespan of the product, etc. These quality properties are quantitative data or numerical. Examples of charts that correspond to this type of data include mean control chart ($\bar{X}$ chart), range control chart (R chart), and standard deviation control chart (S chart), which are considered standard control chart. It was first proposed for use by Shewhart [1] when the process has a normal distribution and is effective in detecting large changes [2]. The shortcoming of standard control charts is that they do not provide accuracy important to historical data. This characteristic is a property of the chart called "memoryless", so it cannot record small changes that occurred in the past, and these small changes accumulate over time. Therefore, several researchers have studied the properties of giving weight to data or consideration of data based on time.

It has proposed a control chart that gives importance to past data by giving the chart the property of being able to remember changes from the past until the present, This property is the ability to remember, making it possible to overcome the disadvantages of the chart can control standards. Because the control is effectively controlled, it will analyze the changes in the data with the average value, which is calculated by averaging various factors in the data. It emphasizes historical data through weighting. For example, the Cumulative Sum Control Chart (CUSUM chart) was first proposed by Page [3]. In 1959, Roberts [4] proposed the Exponentially Weighted Moving Average Control Chart (EWMA chart) with emphasis on historical data with weight decreasing/increasing exponentially. Both control charts have gained widespread popularity due to their ability to detect small process changes [2]. Later, in 1994, Butler and Stefani [5] introduced the Double Exponentially Weighted Moving Average control chart (DEWMA chart), which extended from the EWMA chart to detect small to moderate shifts. A Moving Average Control Chart (MA chart) was created later in 2004 by Khoo [6], utilizing a straightforward concept to compute the MA statistics by providing a span of average (w). The efficiency of this control chart is suitable for small to moderate shifts, and it is simple to calculate and implement [7]–[9]. The MA chart is investigated to monitor not only a change in the process mean but also process dispersion [10]–[12]. Khoo and Wong [13] jointly developed a Double Moving Average Control Chart (DMA chart). DMA chart are effective in detecting changes in the process mean at small change levels. The DMA chart outperformance the MA chart for small shifts in which the explicit formulas of the average run length (ARL) are derived [14], [15]. Recently, Alevizakos *et al*., [16] proposed the triple moving average control chart (TMA chart) to enhance the performance of the DMA chart for detecting a small change in process mean under normal distribution. They also corrected the part of the variance of the DMA chart. Unfortunately, the explicit formulas of the average run length (ARL) of the TMA chart which the explicit formulas of ARL did not yet proven and presented.

Consequently, this research aims to propose the explicit formulas of the ARL of the TMA chart for both cases of normal and non-normal process observations. In addition, the performance of the TMA chart is compared with the DMA, MA and Shewhart charts.

2 Control Charts and Properties

In this research, the time-varying control chart via moving average control charts are investigated as follows: single moving average, double moving average and triple moving average control charts, as well as the Shewhart chart. The performance of the TMA chart is compared with the performance of an MA, DMA and Shewhart charts by considering the average running length when the production process is out of control (ARL_1) as a criterion for comparing efficiency. In this section, therefore, the literature on control charts and related theories are discussed as follows.

2.1 *Moving average control chart (MA chart)*

In 2004, a statistician named Khoo invented the moving average control chart. It is a control chart that can detect small-size parameter changes (small-size are in the range 0 to 1) well, which is a chart that weights historical data using a moving average. It has a movement time of w, which allows for good detection of small to medium changes by definition of chart statistics and control limits. The following statistics define the MA chart:

$$MA_t = \begin{cases} \dfrac{1}{t}\sum_{i=1}^{t} X_i & ; \quad t < w \\ \dfrac{1}{w}\sum_{i=t-w+1}^{t} X_i & ; \quad t \geq w \end{cases}. \tag{1}$$

From Equation (1), it is transformed into a linear equation as follows Equation (2):

$$MA_t = \sum_{i=\max(1,t-w+1)}^{t} a_{i,t}^{MA} \cdot X_i \quad (2)$$

where X_t is the observed value of the process with mean μ and variance σ^2, w is the width of the MA chart and $a_{i,t}^{MA}$ is the constant coefficient of X as following Equation (3), where

$$a_{i,t}^{MA} = \begin{cases} \dfrac{1}{i} & ; \quad t < w \\ \dfrac{1}{w} & ; \quad t \ge w \end{cases}. \quad (3)$$

The mean of the MA chart is

$$E(MA_t) = \mu$$

and its variance is

$$V(MA_t) = \left(\sum_{i=\max(1,t-w+1)}^{t} a_{i,t}^{MA} \right)^2 \sigma^2.$$

The control limits of the MA control chart are given as Equation (4),

$$UCL_i / LCL_i = E(MA_i) \pm L_{MA} \cdot \sqrt{V(MA_i)} \quad (4)$$

where L_{MA} is the control limit coefficient of the MA chart.

2.2 *Double moving average control chart (DMA chart)*

In 2008, Khoo and Wong developed a new control chart. It is based on the MA chart, namely the double moving average chart. It is a control chart that is good at detecting small-size parameter changes (small-size are in the range 0 to 1). It has a movement time equal to w by definition of the chart's statistics and control limits. The following statistics define the DMA chart:

$$DMA_t = \begin{cases} \dfrac{1}{t} \sum_{i=1}^{t} MA_i & ; \quad t < w \\ \dfrac{1}{w} \sum_{i=t-w+1}^{t} MA_i & ; \quad t \ge w \end{cases}. \quad (5)$$

From Equation (5), it is transformed into a linear equation that follows Equation (6):

$$DMA_t = \sum_{i=\max(1,t-2w+2)}^{t} a_{i,t}^{DMA} \cdot X_i \quad (6)$$

where X_t is the observed value of the process with mean μ and variance is σ^2, w is the width of the DMA chart and $a_{i,t}^{DMA}$ is the constant coefficient of X as following Equation (7), where

$$a_{i,t}^{DMA} = \begin{cases} b_{i,t}^1 & ; \quad t < w \\ b_{i,t}^2 & ; \quad w \le t < 2w-1. \\ b_{i,t}^3 & ; \quad t \ge 2w-1 \end{cases} \quad (7)$$

Period $t < w$,

$$b_{i,t}^1 = \frac{1}{t} \sum_{j=i}^{t} \frac{1}{j} \quad ; \quad i \le t.$$

Period $w \le t < 2w-1$,

$$b_{i,t}^2 = \begin{cases} \left(\dfrac{i}{w^2} \right) + \dfrac{1}{w} \sum_{j=t-w+1}^{w-1} \dfrac{1}{j} & ; \quad i < t-w+1 \\ \left(\dfrac{t-w+1}{w^2} \right) + \dfrac{1}{w} \sum_{j=i}^{w-1} \dfrac{1}{j} & ; \quad t-w+1 \le i < w, \\ \dfrac{t+1-i}{w^2} & ; \quad i \ge w \end{cases}$$

and period $t \ge 2w-1$,

$$b_{i,t}^3 = \begin{cases} \dfrac{j-(t-2w+1)}{w^2} & ; \quad i < t-w+2 \\ \dfrac{t+1-j}{w^2} & ; \quad i \ge t-w+2 \end{cases}.$$

The mean of the DMA chart is

$$E\left(DMA_t\right) = \mu$$

and its variance is

$$V\left(DMA_t\right) = \left(\sum_{i=\max(1,t-2w+2)}^{t} a_{i,t}^{DMA}\right)^2 \sigma^2.$$

The control limits of DMA chart are given

$$UCL_i \,/\, LCL_i = E\left(DMA_i\right) \pm L_{DMA} \cdot \sqrt{V\left(DMA_i\right)}$$

where L_{DMA} is the control limit coefficient of the DMA chart.

2.3 *Triple moving average control chart (TMA chart)*

In 2021, statisticians Alevizakos, Chatterjee and Koukouvinos presented three moving average control charts. To develop a DMA chart, the TMA chart has good performance in detecting small parameter changes by taking the statistics of the control chart to average two times. Find the average again one more time, for a total of 3 repeats of the average by the definition of the statistic and control limits. The equation is as follows. The following statistics define the TMA chart:

$$TMA_t = \begin{cases} \dfrac{1}{t}\sum\limits_{i=1}^{t} DMA_i & t < w \\ \dfrac{1}{w}\sum\limits_{i=t-w+1}^{t} DMA_i & t \geq w \end{cases}. \tag{8}$$

From Equation (8), it is transformed into a linear equation that follows Equation (9) :

$$TMA_t = \sum_{i=\max(1,t-3w+3)}^{t} a_{i,t}^{TMA} \cdot X_i \tag{9}$$

where X_t is the observed value of the process with mean μ and variance is σ^2, w is the width of the TMA chart and $a_{i,t}^{TMA}$ is the constant coefficient of X as following Equation (10), where

$$a_{i,t}^{TMA} = \begin{cases} c_{i,t}^1 & ; \quad t < w \\ c_{i,t}^2 & ; \quad w \leq t < 2w-1 \\ c_{i,t}^3 & ; \quad 2w-1 \leq t < 3w-2 \\ c_{i,t}^4 & ; \quad t \geq 3w-2 \end{cases}. \tag{10}$$

Period $t < w$,

$$c_{i,t}^1 = \frac{1}{t}\sum_{j=i}^{t}\left(\frac{1}{j}\sum_{k=j}^{t}\frac{1}{k}\right) \quad ; \quad i \leq t.$$

Period $w \leq t < 2w-1$,

$$c_{i,t}^2 = \begin{cases} \left(\begin{array}{l} \sum\limits_{j=i}^{t-w}\left(\left(\dfrac{1}{w^2}\right) + \dfrac{1}{w \cdot i}\sum\limits_{j=t-w+1}^{w-1}\dfrac{1}{j}\right) \\ + \sum\limits_{j=t-w+1}^{w-1}\left(\left(\dfrac{t-w+1}{w^2 \cdot i}\right) + \dfrac{1}{w \cdot i}\sum\limits_{j=i}^{w-1}\dfrac{1}{j}\right) \\ + \sum\limits_{j=w}^{w-1+i}\left(\dfrac{t+1-i}{w^3}\right) \end{array}\right) & ; \quad i < t-w+1 \\ \left(\begin{array}{l} \sum\limits_{j=i}^{w-1}\left(\left(\dfrac{t-w+1}{w^2 \cdot i}\right) + \dfrac{1}{w \cdot i}\sum\limits_{j=i}^{w-1}\dfrac{1}{j}\right) \\ + \sum\limits_{j=w}^{t}\left(\dfrac{t+1-i}{w^3}\right) \end{array}\right) & ; \quad t-w+1 \leq i < w \\ \sum\limits_{j=i}^{t}\left(\dfrac{t+1-j}{w^3}\right) & ; \quad i \geq w \end{cases}.$$

Period $2w-1 \leq t < 3w-2$,

$$c_{i,t}^3 = \begin{cases} \left(\begin{array}{l} \sum\limits_{j=t-2w+2}^{w-1}\left(\dfrac{i-(t-2w+1)}{i \cdot w^2}\right) \\ + \sum\limits_{j=w}^{i+w-1}\left(\dfrac{i-(t-2w+1)}{w^3}\right) \end{array}\right) & ; \quad i < t-2w+3 \\ \left(\begin{array}{l} \sum\limits_{j=i}^{w-1}\left(\dfrac{i-(t-2w+1)}{i \cdot w^2}\right) \\ + \sum\limits_{j=w}^{t-w+1}\left(\dfrac{i-(t-2w+1)}{w^3}\right) \\ + \sum\limits_{j=t-w+2}^{i+w-1}\left(\dfrac{t+1-i}{w^3}\right) \end{array}\right) & ; \quad t-2w+3 \leq i < w \\ \left(\begin{array}{l} \sum\limits_{j=i}^{t-w+1}\left(\dfrac{i-(t-2w+1)}{w^3}\right) \\ + \sum\limits_{j=t-w+2}^{i+w-1}\left(\dfrac{t+1-i}{w^3}\right) \end{array}\right) & ; \quad w \leq i < t-w+2 \\ \sum\limits_{j=i}^{t}\dfrac{t+1-i}{w^3} & ; \quad i \geq t-w+2 \end{cases}$$

and period $t \geq 3w-2,$

$$c_{i,t}^{4}=\begin{cases}\sum_{j=t-2w+2}^{i+w-1}\left(\frac{i-(t-2w+1)}{w^3}\right) & ; \quad i>t-2w+1\\ \left(\sum_{j=i}^{t-w+1}\left(\frac{i-(t-2w+1)}{w^3}\right)+\sum_{j=t-w+2}^{i+w-1}\left(\frac{t+1-i}{w^3}\right)\right) & ; \quad t-2w+1\leq i<t-w+2.\\ \sum_{j=i}^{t}\left(\frac{t+1-i}{w^3}\right) & ; \quad i\geq t-w+2\end{cases}$$

The mean of the TMA chart is

$$E(TMA_t)=\mu$$

and its variance is

$$V(TMA_t)=\left(\sum_{i=\max(1,t-3w+3)}^{t} a_{i,t}^{TMA}\right)^2\sigma^2.$$

The control limits of the TMA chart is given.

$$UCL_i / LCL_i = E(TMA_i)\pm L_{TMA}\cdot\sqrt{V(TMA_i)}$$

where L_{TMA} is the control limit coefficient of the TMA chart.

3 Research Results

In this section, the explicit formula for the average run length of the TMA chart will be presented, and its performance will be compared with the MA and DMA charts.

3.1 *Explicit formula for average run length*

To create a formula for calculating the ARL of a TMA chart, which is the average of the run length (RL), where RL is the number of process observations that are in control before the process observation values change. Not under control, which RL has a geometric distribution with parameter *p* is the probability that the observed value will not be under control (out of control: o.o.c) of the TMA chart by constructing a formula for calculating the ARL of the TMA chart with the following details:

Given,

$ARL=n$ and $\Pr(\text{o.o.c. at time } i\leq ARL)=1$.

Let $1=\Pr(\text{o.o.c. at time } i\leq ARL)$

$$\begin{aligned}&=\sum_{i=1}^{w-1}\left(\Pr(TMA_i\geq UCL_i)+\Pr(TMA_i\leq LCL_i)\right)\\&+\sum_{i=w}^{2w-2}\left(\Pr(TMA_i\geq UCL_i)+\Pr(TMA_i\leq LCL_i)\right)\\&+\sum_{i=2w-1}^{3w-3}\left(\Pr(TMA_i\geq UCL_i)+\Pr(TMA_i\leq LCL_i)\right)\\&+(n-(3w-3))\begin{pmatrix}\Pr(TMA_{3w-2}\geq UCL_{3w-2})\\+\Pr(TMA_{3w-2}\leq LCL_{3w-2})\end{pmatrix}.\end{aligned}$$

So,

$$1=A+B+C+(n-(3w-3))D.$$

From Equation (11), it can be seen that the formula for calculating the ARL of the TMA control chart is

$$ARL=\frac{1-(A+B+C)}{D}+(3w-3) \tag{11}$$

where

$$A=\sum_{i=1}^{w-1}\begin{pmatrix}\Pr\left(Z\geq\frac{\left(\mu_0+L_{TMA}\cdot\sqrt{v_i\cdot\sigma_0^2}\right)-\mu_1}{\sqrt{v_i\cdot\sigma_1^2}}\right)\\+\Pr\left(Z\leq\frac{\left(\mu_0-L_{TMA}\cdot\sqrt{v_i\cdot\sigma_0^2}\right)-\mu_1}{\sqrt{v_i\cdot\sigma_1^2}}\right)\end{pmatrix}$$

$$B=\sum_{i=w}^{2w-2}\begin{pmatrix}\Pr\left(Z\geq\frac{\left(\mu_0+L_{TMA}\cdot\sqrt{v_i\cdot\sigma_0^2}\right)-\mu_1}{\sqrt{v_i\cdot\sigma_1^2}}\right)\\+\Pr\left(Z\leq\frac{\left(\mu_0-L_{TMA}\cdot\sqrt{v_i\cdot\sigma_0^2}\right)-\mu_1}{\sqrt{v_i\cdot\sigma_1^2}}\right)\end{pmatrix}$$

$$C=\sum_{i=2w-1}^{3w-3}\begin{pmatrix}\Pr\left(Z\geq\frac{\left(\mu_0+L_{TMA}\cdot\sqrt{v_i\cdot\sigma_0^2}\right)-\mu_1}{\sqrt{v_i\cdot\sigma_1^2}}\right)\\+\Pr\left(Z\leq\frac{\left(\mu_0-L_{TMA}\cdot\sqrt{v_i\cdot\sigma_0^2}\right)-\mu_1}{\sqrt{v_i\cdot\sigma_1^2}}\right)\end{pmatrix}$$

$$\begin{aligned}D&=\Pr\left(Z\geq\frac{\left(\mu_0+L_{TMA}\cdot\sqrt{v_{3w-2}\cdot\sigma_0^2}\right)-\mu_1}{\sqrt{v_{3w-2}\cdot\sigma_1^2}}\right)\\&+\Pr\left(Z\leq\frac{\left(\mu_0-L_{TMA}\cdot\sqrt{v_{3w-2}\cdot\sigma_0^2}\right)-\mu_1}{\sqrt{v_{3w-2}\cdot\sigma_1^2}}\right)\end{aligned}$$

where μ_0 and σ_0^2 are mean and variance of process at not change on parameter, μ_1 and σ_1^2 are mean and variance of process at change on parameter, w is the width of the TMA chart, $v_i = \left(a_{i,t}^{TMA}\right)^2$ is the constant coefficient of variance of X as following Equation (12), where

$$v_i = \left(a_{i,t}^{TMA}\right)^2 \tag{12}$$

and Z is cumulative probability of the standard normal distribution.

3.2 *Explicit formula for limit of control*

From the explicit formula Equation (11) of ARL of the TMA chart create formulas L_{TMA} with the following details by given $\delta = 0$ and $n = ARL_0$.
Let,

$$A+B+C = \sum_{i=1}^{3w-3}\left(\begin{aligned}&\Pr\left(Z \geq \frac{\left(\mu_0 + L_{TMA}\cdot\sqrt{v_i\cdot\sigma_0^2}\right)-\mu_0}{\sqrt{v_i\cdot\sigma_0^2}}\right)\\ &+\Pr\left(Z \leq \frac{\left(\mu_0 - L_{TMA}\cdot\sqrt{v_i\cdot\sigma_0^2}\right)-\mu_0}{\sqrt{v_i\cdot\sigma_0^2}}\right)\end{aligned}\right)$$

$$\begin{aligned}&= \sum_{i=1}^{3w-3}\left(\Pr\left(Z \geq L_{TMA}\right)+\Pr\left(Z \leq -L_{TMA}\right)\right)\\ &= \sum_{i=1}^{3w-3}\left(\left(1-\Pr\left(Z \leq L_{TMA}\right)\right)-\Pr\left(Z \leq L_{TMA}\right)\right)\\ &= \sum_{i=1}^{3w-3}\left(1-2\Pr\left(Z \leq L_{TMA}\right)\right)\\ &= \left(3w-3\right)\left(1-2\Pr\left(Z \leq L_{TMA}\right)\right)\end{aligned}$$

and

$$\begin{aligned}D &= \Pr\left(Z \geq \frac{\left(\mu_0 + L_{TMA}\cdot\sqrt{a_{3w-2}\cdot\sigma_0^2}\right)-\mu_0}{\sqrt{a_{3w-2}\cdot\sigma_1^2}}\right)\\ &\quad+\Pr\left(Z \leq \frac{\left(\mu_0 - L_{TMA}\cdot\sqrt{a_{3w-2}\cdot\sigma_0^2}\right)-\mu_0}{\sqrt{a_{3w-2}\cdot\sigma_1^2}}\right)\\ &= 1-2\Pr\left(Z \leq L_{TMA}\right).\end{aligned}$$

So,

$$\begin{aligned}1 &= \left(3w-3\right)\left(1-2\Pr\left(Z \leq L_{TMA}\right)\right)\\ &\quad+\left(n-\left(3w-3\right)\right)\left(1-2\Pr\left(Z \leq L_{TMA}\right)\right)\\ 1 &= n\left(1-2\Pr\left(Z \leq L_{TMA}\right)\right).\end{aligned}$$

$$\begin{aligned}\Pr\left(Z \leq L_{TMA}\right) &= \frac{1}{2}\left(1-\frac{1}{n}\right)\\ &= \frac{ARL_0 - 1}{2ARL_0}.\end{aligned}$$

Let,

$$L_{TMA} = Z_\alpha \tag{13}$$

where α is the cumulative probability of the standard normal distribution from Equation (13). It is equal to Equation (14) as follows

$$\alpha = \frac{ARL_0 - 1}{2ARL_0} \tag{14}$$

where Z_α is the statistical value of the cumulative probability from a standard normal distribution which is obtained from the cumulative standard normal table (Table Z)

From the above formula, the control limit value for the TMA chart can be obtained as follows:

1) when $ARL_0 = 200,\ L = 2.8070$
2) when $ARL_0 = 370,\ L = 2.9997$
3) when $ARL_0 = 500,\ L = 3.0902$

3.3 *Performance results*

To compare the performance of the MA, DMA, and TMA charts for this topic, use the ARL values calculated and a lower chart's ARL value means that the chart is the best performance, with the equations and determine the distribution of the observed values in the process with a symmetric distribution, i.e., a normal distribution, logistics distribution and Laplace distribution. The results of the analysis of the efficiency of the control chart are as follows:

For the process, the data is normally distributed. It was found that the TMA chart is superior to other control charts *w* = 2 and 3, for all magnitude of shift in process mean. When *w* = 4, the TMA chart plays a vital role in detecting small to moderate shifts when $\delta \in [0.1, 0.8]$, when *w* = 5, the performance of the TMA chart is better than other control charts to detect in the range of $\delta \in [0.1, 0.6]$. In addition, the numerical results found that as the span value (*w*) increases, the TMA control chart's ability to detect small mean changes decreases as following results; when *w* = 10, the TMA chart performed best when the change value is from 0.1 to 0.3 and when *w* = 15, the TMA chart has the best performance when the change value occurs from 0.1 to 0.2.

For the process of data being distributed logistically, it was found that

1) When *w* is equal to 2, the TMA control chart performed best when the change value was from 0 to 2, because ARL value in change is min.

2) When *w* is equal to 3, the TMA control chart performed best when the change value occurred from 0 to 1.

3) When *w* was equal to 4, the TMA control chart performed best when the change value was from 0 to 1.

4) When *w* is equal to 5, it was found that the TMA control chart has the best performance when the change value occurs from 0 to 1.

5) When *w* was equal to 10, the TMA control chart performed best when the change value was from 0 to 0.6.

6) When *w* was equal to 15, the TMA control chart performed best when the change value was from 0 to 0.4.

For the data process with a Laplace distribution, it was found that when *w* is equal to 2 and 3, the TMA chart shows the best performance when the δ is between 0.1 to 2, because of minimum ARL_1. For the case of *w* =4, the performance of the TMA chart is superior to other control charts when δ varies from 0.1 to 1. For other cases, the small change detection performance of the TMA control chart decreases with increasing *w* value as follows: when *w* = 5, the TMA chart has the best performance when $\delta \in [0.1, 0.9]$, when *w* = 10, the TMA chart shows the best performance when $\delta \in [0.1, 0.4]$, and when *w*=15, the TMA chart performed best when $\delta \in [0.1, 0.3]$,

From the results analysis, it is found that the efficiency of the TMA chart when the value of span (*w*) is low for detecting small changes in the change range from 0 to 1. However, when the value *w* is higher, it is found that the effectiveness of the TMA chart in detecting changes has decreased for all case studies (Tables 1–6).

4 Application

In the application of the TMA control chart with the data in Table 7, the process has a mean of 1.5 and a variance of 0.01. The control chart used to detect changes for this data is set to *w = 3* and *L = 3.3* found that the MA and DMA charts can detect process mean changes at an observation value of 20. While the TMA chart can early detect mean shift at an observation value of 19. The TMA chart is more sensitive to monitor than the former chart. The analysis results reveal that TMA control charts are more sensitive to detecting changes than MA and DMA control charts (Figure 1 and Table 8).

Table 1: ARLs of MA, DMA and TMA charts when ARL_0 = 200 for the Normal(0,1) process.

w	Chart	δ											
		0	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**	**2**
2	MA	200.00	183.89	147.21	108.84	78.00	55.68	40.11	29.33	21.82	16.54	12.78	2.55
	DMA	200.00	179.09	135.03	93.71	63.66	43.60	30.48	21.85	16.10	12.20	9.51	2.59
	TMA	200.00	175.44	126.62	84.22	55.37	37.08	25.58	18.27	13.52	10.38	8.25	2.83
3	MA	200.00	176.75	129.53	87.36	57.97	38.99	26.89	19.09	13.99	10.58	8.25	2.38
	DMA	200.00	168.55	112.40	69.68	43.57	28.30	19.28	13.82	10.42	8.24	6.80	2.93
	TMA	200.00	163.19	102.76	60.99	37.29	24.15	16.70	12.34	9.70	8.03	6.94	3.59
4	MA	200.00	170.13	115.44	72.52	45.66	29.65	20.06	14.19	10.50	8.12	6.53	2.48
	DMA	200.00	159.44	96.50	55.59	33.39	21.48	14.90	11.12	8.86	7.44	6.52	3.25
	TMA	200.00	153.06	87.14	48.56	29.12	19.24	14.00	11.08	9.36	8.29	7.57	4.20
5	MA	200.00	164.00	103.98	61.78	37.51	23.92	16.15	11.58	8.79	7.03	5.87	2.59
	DMA	200.00	151.43	84.73	46.54	27.59	18.07	13.06	10.28	8.64	7.62	6.93	3.45
	TMA	200.00	144.40	76.14	41.12	24.99	17.30	13.41	11.30	10.07	9.27	8.69	4.68
10	MA	200.00	139.03	69.29	35.75	20.95	14.09	10.68	8.83	7.74	7.00	6.44	2.73
	DMA	200.00	122.32	55.02	29.56	19.95	15.93	13.99	12.86	12.02	11.26	10.49	3.73
	TMA	200.00	115.06	51.68	30.40	22.95	19.94	18.44	17.43	16.56	15.66	14.69	6.08
15	MA	200.00	121.03	52.83	27.06	17.31	13.23	11.25	10.10	9.26	8.52	7.79	2.73
	DMA	200.00	104.61	45.14	27.72	22.08	19.78	18.42	17.23	15.97	14.57	13.07	3.75
	TMA	200.00	99.51	46.74	33.13	28.97	27.14	25.79	24.38	22.76	20.99	19.19	6.68

Table 2: ARLs of MA, DMA and TMA charts when ARL_0 = 200 for the Logistic(0,1) process.

w	Chart	δ											
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2
2	MA	200.00	194.84	180.73	160.94	139.03	117.69	98.44	81.84	67.92	56.43	47.01	10.02
	DMA	200.00	193.20	175.11	150.97	125.84	102.84	83.26	67.25	54.43	44.25	36.20	7.59
	TMA	200.00	191.91	170.86	143.84	116.91	93.32	74.00	58.73	46.85	37.66	30.55	6.75
3	MA	200.00	192.37	172.38	146.34	119.97	96.50	77.02	61.43	49.17	39.60	32.13	6.60
	DMA	200.00	189.38	162.94	131.30	102.15	78.40	60.16	46.49	36.33	28.77	23.11	5.80
	TMA	200.00	187.32	156.86	122.38	92.41	69.23	52.19	39.86	30.98	24.54	19.83	6.18
4	MA	200.00	189.97	164.75	134.06	105.25	81.39	62.79	48.69	38.09	30.14	24.15	5.43
	DMA	200.00	185.83	152.64	116.46	86.16	63.49	47.25	35.77	27.64	21.84	17.65	5.86
	TMA	200.00	183.18	145.54	107.17	77.07	55.74	41.11	31.14	24.29	19.52	16.17	7.05
5	MA	200.00	187.64	157.76	123.60	93.58	70.16	52.79	40.16	31.00	24.32	19.43	5.07
	DMA	200.00	182.49	143.74	104.81	74.70	53.59	39.26	29.55	22.92	18.34	15.13	6.41
	TMA	200.00	179.35	136.06	95.75	66.65	47.35	34.80	26.61	21.18	17.52	15.00	8.22
10	MA	200.00	176.91	130.22	88.76	60.04	41.60	29.90	22.41	17.53	14.28	12.07	5.95
	DMA	200.00	168.11	112.68	71.83	47.54	33.65	25.62	20.84	17.91	16.04	14.79	9.66
	TMA	200.00	163.52	105.41	66.63	45.25	33.70	27.30	23.63	21.42	20.02	19.07	13.65
15	MA	200.00	167.55	111.25	69.85	45.26	31.20	23.07	18.22	15.24	13.34	12.06	7.03
	DMA	200.00	156.73	94.93	58.24	39.69	30.30	25.35	22.59	20.94	19.84	19.03	11.54
	TMA	200.00	151.94	90.38	57.65	42.36	35.06	31.37	29.34	28.10	27.20	26.44	17.42

Table 3: ARLs of MA, DMA and TMA charts when ARL_0 = 200 for the Laplace(0,1) process.

w	Chart	δ											
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2
2	MA	200.00	191.64	169.98	142.35	115.01	91.21	71.85	56.62	44.82	35.72	28.70	5.38
	DMA	200.00	189.02	161.83	129.54	100.04	76.20	57.99	44.42	34.37	26.92	21.36	4.47
	TMA	200.00	186.99	155.85	120.81	90.51	67.20	50.12	37.81	28.95	22.54	17.86	4.35
3	MA	200.00	187.72	157.97	123.82	93.69	70.10	52.55	39.75	30.45	23.65	18.66	3.97
	DMA	200.00	183.05	145.05	106.23	75.69	53.99	39.08	28.89	21.87	16.98	13.53	4.19
	TMA	200.00	179.88	137.11	96.50	66.65	46.57	33.36	24.64	18.81	14.85	12.11	4.90
4	MA	200.00	183.97	147.47	109.31	78.64	56.45	40.97	30.27	22.83	17.59	13.87	3.68
	DMA	200.00	177.62	131.74	90.23	61.01	41.99	29.80	21.93	16.75	13.29	10.92	4.68
	TMA	200.00	173.64	123.05	81.01	53.47	36.51	26.09	19.60	15.45	12.74	10.93	5.94
5	MA	200.00	180.37	138.24	97.70	67.54	47.05	33.46	24.43	18.36	14.21	11.34	3.75
	DMA	200.00	172.62	120.85	78.62	51.36	34.77	24.67	18.41	14.44	11.86	10.13	5.27
	TMA	200.00	168.01	111.99	70.34	45.37	30.98	22.59	17.57	14.48	12.50	11.19	6.96
10	MA	200.00	164.45	105.23	63.62	39.73	26.39	18.79	14.33	11.61	9.88	8.74	4.61
	DMA	200.00	152.22	87.20	50.39	32.33	23.34	18.65	16.06	14.52	13.52	12.79	7.08
	TMA	200.00	146.20	80.83	47.69	32.62	25.54	21.98	20.05	18.87	18.04	17.37	10.54
15	MA	200.00	151.40	85.43	48.15	29.86	20.76	16.00	13.37	11.80	10.77	10.03	4.90
	DMA	200.00	137.43	71.25	41.74	29.45	24.01	21.36	19.86	18.84	17.99	17.15	7.64
	TMA	200.00	131.86	68.91	44.00	34.42	30.39	28.42	27.21	26.25	25.30	24.27	12.81

Table 4: ARLs of MA, DMA and TMA charts when ARL_0 = 370 for the Normal(0,1) process.

w	Chart	δ											
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	2
2	MA	370.00	336.57	262.62	188.41	131.09	90.98	63.79	45.43	32.93	24.32	18.32	2.95
	DMA	370.00	326.67	238.58	159.79	104.90	69.52	47.05	32.66	23.29	17.09	12.90	2.86
	TMA	370.00	319.15	222.09	142.00	89.88	57.99	38.54	26.50	18.89	13.96	10.70	3.08
3	MA	370.00	321.89	227.89	148.03	94.77	61.58	41.04	28.16	19.93	14.56	10.99	2.60
	DMA	370.00	305.05	194.58	115.19	68.87	42.75	27.81	19.03	13.72	10.40	8.27	3.18
	TMA	370.00	294.03	175.88	99.03	57.41	35.20	23.03	16.13	12.06	9.58	8.00	3.89
4	MA	370.00	308.36	200.59	120.64	72.83	45.37	29.43	19.96	14.16	10.51	8.16	2.68
	DMA	370.00	286.48	164.10	89.38	50.73	30.78	20.13	14.21	10.78	8.71	7.40	3.56
	TMA	370.00	273.32	145.92	76.06	42.55	26.18	17.81	13.31	10.76	9.24	8.27	4.60
5	MA	370.00	295.85	178.64	101.03	58.44	35.48	22.80	15.56	11.27	8.64	6.96	2.80
	DMA	370.00	270.18	141.68	72.81	40.28	24.58	16.61	12.36	9.95	8.51	7.59	3.82
	TMA	370.00	255.61	124.81	62.11	34.65	22.11	16.01	12.84	11.07	10.00	9.28	5.16

Table 4 *(continued)*.

w	Chart	δ											
		0	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**	**2**
10	MA	370.00	245.45	113.23	53.92	29.06	18.04	12.79	10.09	8.58	7.63	6.97	3.00
	DMA	370.00	211.00	84.69	40.26	24.38	18.09	15.26	13.76	12.78	11.98	11.22	4.20
	TMA	370.00	194.89	75.74	38.59	26.30	21.64	19.54	18.32	17.38	16.49	15.56	6.88
15	MA	370.00	209.40	82.25	37.61	21.66	15.32	12.46	10.95	9.96	9.16	8.42	3.01
	DMA	370.00	174.35	63.75	33.70	24.58	21.20	19.50	18.25	17.01	15.67	14.21	4.24
	TMA	370.00	160.66	61.47	37.73	31.01	28.46	26.93	25.54	24.00	22.30	20.53	7.67

Table 5: ARLs of MA, DMA and TMA charts when ARL_0 = 370 for the Logistic(0,1) process.

w	Chart	δ											
		0	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**	**2**
2	MA	370.00	359.24	330.09	289.96	246.53	205.25	168.84	138.11	112.83	92.31	75.76	13.98
	DMA	370.00	355.80	318.53	269.96	220.73	176.88	140.48	111.36	88.48	70.65	56.78	9.96
	TMA	370.00	353.09	309.84	255.73	203.40	158.85	123.31	95.85	74.90	58.98	46.87	8.46
3	MA	370.00	354.09	312.99	260.80	209.45	165.02	129.06	100.93	79.25	62.62	49.87	8.51
	DMA	370.00	347.82	293.71	231.00	175.15	131.06	98.09	73.95	56.37	43.54	34.11	6.83
	TMA	370.00	343.48	281.28	213.36	156.46	113.86	83.37	61.85	46.66	35.86	28.10	6.94
4	MA	370.00	349.08	297.47	236.54	181.23	136.81	103.10	78.12	59.74	46.20	36.18	6.56
	DMA	370.00	340.40	272.83	201.95	144.84	103.48	74.67	54.78	41.00	31.36	24.54	6.51
	TMA	370.00	334.77	258.30	183.60	127.33	88.77	63.07	45.96	34.47	26.65	21.24	7.59
5	MA	370.00	344.22	283.31	216.09	159.14	116.10	85.06	63.00	47.35	36.16	28.10	5.84
	DMA	370.00	333.40	254.85	179.32	123.23	85.24	60.15	43.57	32.51	25.02	19.87	6.94
	TMA	370.00	326.68	239.07	161.29	107.48	73.00	51.22	37.35	28.37	22.45	18.47	8.73
10	MA	370.00	321.83	228.14	149.02	96.55	64.00	43.96	31.46	23.50	18.34	14.90	6.43
	DMA	370.00	303.02	192.24	115.18	71.39	47.22	33.66	25.82	21.15	18.25	16.39	10.41
	TMA	370.00	292.66	176.09	102.86	64.29	44.18	33.39	27.38	23.89	21.76	20.39	14.56
15	MA	370.00	302.27	190.49	112.91	68.89	44.61	30.98	23.10	18.40	15.48	13.61	7.66
	DMA	370.00	278.52	155.70	87.25	54.18	38.02	29.78	25.36	22.84	21.29	20.23	12.67
	TMA	370.00	266.67	142.93	81.25	53.69	41.00	34.81	31.58	29.73	28.53	27.63	18.77

Table 6: ARLs of MA, DMA and TMA charts when ARL_0 = 370 for the Laplace(0,1) process.

w	Chart	δ											
		0	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**	**2**
2	MA	370.00	352.58	308.19	253.05	200.12	155.39	119.92	92.65	71.95	56.27	44.38	6.94
	DMA	370.00	347.13	291.65	227.90	171.63	127.55	94.80	70.94	53.65	41.07	31.85	5.39
	TMA	370.00	342.89	279.58	210.87	153.64	111.01	80.62	59.24	44.18	33.49	25.84	5.02
3	MA	370.00	344.44	283.91	216.83	159.77	116.50	85.19	62.89	47.02	35.65	27.45	4.70
	DMA	370.00	334.72	257.99	182.85	126.11	87.13	61.10	43.74	32.06	24.08	18.56	4.64
	TMA	370.00	328.10	242.06	164.11	109.20	73.51	50.70	36.02	26.44	20.07	15.77	5.29
4	MA	370.00	336.67	262.91	188.92	131.77	91.80	64.71	46.42	33.98	25.42	19.45	4.14
	DMA	370.00	323.45	231.54	152.40	99.04	65.51	44.64	31.51	23.09	17.58	13.91	5.05
	TMA	370.00	315.09	214.12	134.58	84.74	55.12	37.48	26.77	20.11	15.86	13.08	6.34
5	MA	370.00	329.22	244.59	166.83	111.37	74.97	51.52	36.33	26.36	19.70	15.17	4.11
	DMA	370.00	313.06	210.03	130.42	81.29	52.43	35.40	25.13	18.79	14.76	12.14	5.67
	TMA	370.00	303.31	192.18	114.18	69.52	44.71	30.69	22.54	17.65	14.62	12.68	7.44
10	MA	370.00	296.36	179.98	102.97	60.77	38.06	25.55	18.41	14.20	11.61	9.96	5.03
	DMA	370.00	270.54	143.72	76.44	44.96	29.89	22.32	18.30	16.02	14.62	13.68	7.82
	TMA	370.00	257.07	129.22	68.61	42.35	30.48	24.76	21.79	20.11	19.03	18.24	11.47
15	MA	370.00	269.47	141.62	73.96	42.33	27.19	19.58	15.53	13.22	11.81	10.86	5.43
	DMA	370.00	239.18	111.10	57.76	36.60	27.63	23.47	21.31	20.00	19.02	18.16	8.57
	TMA	370.00	225.24	102.11	56.59	39.91	33.23	30.19	28.55	27.42	26.43	25.43	14.10

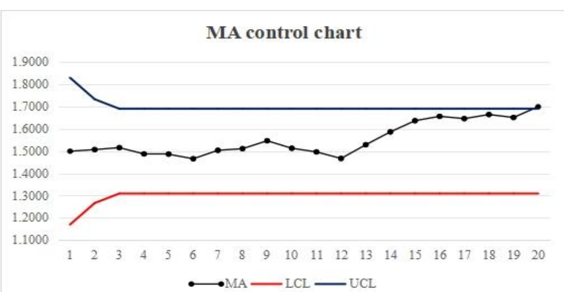
Table 7: Real application data.

Sample	Subgroup				
	1	2	3	4	5
1	1.4483	1.5458	1.4538	1.4303	1.6206
2	1.5435	1.6899	1.583	1.3358	1.4187
3	1.5175	1.3446	1.4723	1.6657	1.6661
4	1.5454	1.0931	1.4072	1.5039	1.5264
5	1.4418	1.5059	1.5124	1.462	1.6263
6	1.4301	1.2725	1.5945	1.5397	1.5252
7	1.4981	1.4506	1.6174	1.5837	1.4962
8	1.3009	1.506	1.6231	1.5831	1.6454
9	1.4132	1.4603	1.5808	1.7111	1.7313
10	1.3817	1.3135	1.4953	1.4894	1.4596
11	1.5765	1.7014	1.4026	1.2773	1.4541
12	1.4936	1.4373	1.5139	1.4808	1.5293
13	1.5729	1.6738	1.5048	1.5651	1.7473
14	1.8089	1.5513	1.825	1.4389	1.6558
15	1.6236	1.5393	1.6738	1.8698	1.5036
16	1.412	1.7931	1.7345	1.6391	1.7791
17	1.7372	1.5663	1.491	1.7809	1.5504
18	1.5971	1.7394	1.6832	1.6677	1.7974
19	1.4295	1.6536	1.9134	1.7272	1.437
20	1.6217	1.822	1.7915	1.6744	1.9404

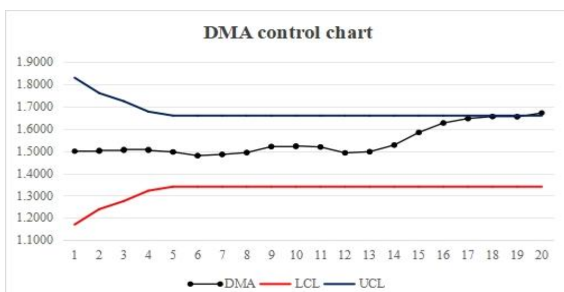
Table 8: Analytical results of real data with MA, DMA amd TMA charts with $w = 3$ and $L = 3.3$

Sample	Data	Control Charts								
		Moving Average			Double Moving Average			Triple Moving Average		
		Statistic	LCL	UCL	Statistic	LCL	UCL	Statistic	LCL	UCL
1	1.4998	1.4998	1.1700	1.8300	1.4998	1.1700	1.8300	1.4998	1.1700	1.8300
2	1.5142	1.5070	1.2667	1.7333	1.5034	1.2391	1.7609	1.5016	1.2083	1.7917
3	1.5332	1.5157	1.3095	1.6905	1.5075	1.2755	1.7245	1.5035	1.2336	1.7664
4	1.4152	1.4875	1.3095	1.6905	1.5034	1.3223	1.6777	1.5048	1.2899	1.7101
5	1.5097	1.4860	1.3095	1.6905	1.4964	1.3402	1.6598	1.5024	1.3311	1.6689
6	1.4724	1.4658	1.3095	1.6905	1.4798	1.3402	1.6598	1.4932	1.3506	1.6494
7	1.5292	1.5038	1.3095	1.6905	1.4852	1.3402	1.6598	1.4871	1.3549	1.6451
8	1.5317	1.5111	1.3095	1.6905	1.4935	1.3402	1.6598	1.4862	1.3549	1.6451
9	1.5793	1.5467	1.3095	1.6905	1.5205	1.3402	1.6598	1.4998	1.3549	1.6451
10	1.4279	1.5130	1.3095	1.6905	1.5236	1.3402	1.6598	1.5126	1.3549	1.6451
11	1.4824	1.4965	1.3095	1.6905	1.5188	1.3402	1.6598	1.5210	1.3549	1.6451
12	1.4910	1.4671	1.3095	1.6905	1.4922	1.3402	1.6598	1.5115	1.3549	1.6451
13	1.6128	1.5287	1.3095	1.6905	1.4974	1.3402	1.6598	1.5028	1.3549	1.6451
14	1.6560	1.5866	1.3095	1.6905	1.5275	1.3402	1.6598	1.5057	1.3549	1.6451
15	1.6420	1.6369	1.3095	1.6905	1.5841	1.3402	1.6598	1.5363	1.3549	1.6451
16	1.6716	1.6565	1.3095	1.6905	1.6267	1.3402	1.6598	1.5794	1.3549	1.6451
17	1.6252	1.6462	1.3095	1.6905	1.6466	1.3402	1.6598	1.6191	1.3549	1.6451
18	1.6970	1.6646	1.3095	1.6905	1.6558	1.3402	1.6598	1.6430	1.3549	1.6451
19	1.6321	1.6514	1.3095	1.6905	1.6541	1.3402	1.6598	**1.6521**	1.3549	1.6451
20	1.7700	1.6997	1.3095	1.6905	1.6719	1.3402	1.6598	1.6606	1.3549	1.6451

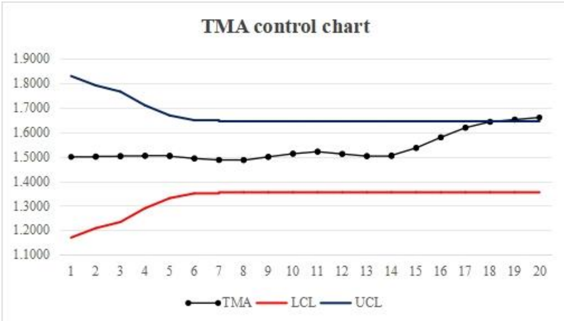
Bold number is the first time signal to out of control.



(a) Moving average control chart



(b) Double moving average control chart



(c) Triple moving average control chart

Figure 1: Performance of control charts from real data with MA, DMA and TMA control charts.

5 Conclusions

The results from ARL formula show that the effectiveness of a TMA chart depends on the value. When the value *w* is low, the performance of the TMA chart is best at detecting small changes. Moreover, when the value *w* is higher, the performance of the TMA chart is best. The TMA chart is less effective at detecting large changes. Compared with MA and DMA charts, and found that when the higher the value, the better the MA chart is at detecting changes. This makes it evident that TMA chart is appropriate for jobs with limited data, requiring them to have low values. However, if the data is large, choosing to use the TMA chart to detect changes that occur should be set. It has a low value. Therefore, when working with data, you should use a variety of control charts to detect changes that occur comprehensively. The research results show that in addition to the value that affects the performance of the TMA chart when *w* is small or large, finding multiple moving averages also results in the control charts in the Moving Average family (MA, DMA, and TMA) being better at detecting small changes.

The formula for success obtained from research shows that in calculating and comparing the performance of control charts, TMA chart can be calculated accurately and quickly, which can be represented and calculated immediately, including being able to calculate control limits without having to make estimates or calculate values from data simulations and create a formula for success in this research. However, the formula in this research is approximated with a standard normal distribution, which may result in non-normally distributed when measuring the efficiency of the process. There can be discrepancies.

The formula in the research, it possible to provide a guideline for calculating the average run length of MA and DMA charts, and to see the pattern of successful formulas for multiple average control charts (4 or more). In the research, the average run length is estimated by the standard normal distribution. It means that the process is estimated by normal processing, so the average run length must be calculated by exact distribution for calculating.

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Author Contributions

A.K.: data curation, data analysis, investigation, methodology, writing an original draft; Y.A.: conceptualization, and project administration and S.S.: research design, conceptualization, investigation, reviewing and editing; funding acquisition. All authors have read and agreed to the published version of the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

References

[1] W. A. Shewhart, *Economic Control of Quality of manufactured Product*. New York: D. Van Nostrand Company, Inc. 1931.

[2] D. C. Montgomery, *Introduction to Statistical Quality Control Case Study*, 6th ed., New York: John Wiley and Sons, 2009.

[3] E. S. Page, "Continuous inspection schemes," *Biometrika*, vol. 41, pp. 100–144, 1954.

[4] S. W. Roberts, "Control chart tests based on geometric moving average," *Technometrics*, vol. 1, pp. 239–250, 1959.

[5] S. W. Butler and J. A. Stefani, "Supervisory run-to-run control of a polysilicon gate etch using in situ ellipsometry," *IEEE Transaction on Semiconductor Manufacturing*, vol. 7, pp. 193–201, 1994.

[6] B. C. Khoo, "A moving average control chart for monitoring the fraction non-conforming," *Quality and Reliability Engineering International*, vol. 20, no. 6, pp. 617–635, Oct. 2004, doi: 10.1002/qre.576.

[7] Y. Areepong and S. Sukparungsee, "The closed-form formulas of average run length of moving average control chart for nonconforming for zero-inflated process," *Far East Journal of Mathematical Sciences*, vol. 75, pp. 385–400, 2013.

[8] C. Chananet, Y. Areepong, and S. Sukparungsee, "An approximate formula for ARL in moving average chart with ZINB data," *Thailand Statistician*, vol. 13, pp. 209–222, 2015.

[9] S. Sukparungsee and S. Phantu, "Explicit formulas of moving average control chart for zero modified geometric integer valued autoregressive process," *Applied Science and Engineering Progress*, vol. 17, no. 1, 2024, Art. no. 6921, doi: 10.14416/j.asep.2023.09.004.

[10] A. A. Olatunde and J. O. Olaomi, "A moving average S control chart for monitoring process variability," *Quality Engineering*, vol. 28, no. 2, pp. 212–219, 2016, doi: 10.1080/08982112.2015.

[11] I. S. Trianntafyllou and M. Ram, *Statistical Modeling of Reliability Structures and Industrial Processes*. Florida: CRC Press, 2022.

[12] S. Sukparungsee, N. Saengsura, Y. Areepong, and S. Phantu, "Mixed tukey-double moving average for monitoring of process mean," *Thailand Statistician*, vol. 19, no. 4, pp. 855–865, 2021.

[13] M. B. C. Khoo and V. H. Wong, "A double moving average control chart," *Communication in Statistics – Simulation and Computation*, vol. 37, pp. 1696–1708, 2008, doi: 10.1080/03610910701832459.

[14] Y. Areepong and S. Sukparungsee, "The closed-form formulas of average run length of moving average control chart for nonconforming for zero-inflated process," *Far East Journal of Mathematical Sciences*, vol. 75, pp. 385–400, 2013.

[15] S. Phantu, Y. Areepong, and S. Sukparungsee, "Average run length formulas for mixed double moving average - exponentially weighted moving average control chart," *Science and Technology Asia*, vol. 28, no.1, pp.77–89, 2023.

[16] V. Alevizakos, K. Chatterjee, and C. Koukouvinos, "The triple exponentially weighted moving average control chart," *Quality Technology and Quantitative Management*, vol. 18, no. 3, pp. 326–354, 2020, doi: 10.1080/16843703.2020.1809063.