บทความวิจัย

ช่วงความเชื่อมั่นสำหรับอัตราส่วนค่ามัธยฐานของประชากรสองกลุ่มโดยวิธีของ Price และ Bonett ร่วมกับวิธีบูตสแทร็ปที

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บทคัดย่อ

งานวิจัยนี้มีวัตถุประสงค์เพื่อนำเสนอช่วงความเชื่อมั่นสำหรับอัตราส่วนค่ามัธยฐานของประชากรสองกลุ่ม และเปรียบเทียบประสิทธิภาพระหว่างวิธีของ Price และ Bonett และวิธีที่เสนอในการวิจัยนี้คือ วิธีของ Price และ Bonett ที่ปรับร่วมกับวิธีบูตสแทร็ปที และวิธีบูตสแทร็ปเปอร์เซ็นต์ไทล์ ศึกษาโดยการจำลองข้อมูลจากโปรแกรม R ใช้เทคนิคมอนติคาร์โล 5,000 ครั้ง ในกรณีคำนวณช่วงความเชื่อมั่นสำหรับอัตราส่วนค่ามัธยฐานของประชากรสองกลุ่มโดยใช้วิธีบูตสแทร็ป ใช้การบูตสแทร็ป 5,000 ครั้ง ศึกษาข้อมูลทั้งสองกลุ่มที่ไม่มีการแจกแจงปกติ และตัวอย่างมีขนาดเล็ก ผลการวิจัยชี้ให้เห็นว่าช่วงความเชื่อมั่นสำหรับอัตราส่วนค่ามัธยฐานของประชากรสองกลุ่มวิธีของ Price และ Bonett ที่ปรับร่วมกับวิธีบูตสแทร็ปที และวิธีบูตสแทร็ปเปอร์เซ็นต์ไทล์ มีประสิทธิภาพที่ดีกว่าวิธีของ Price และ Bonett 18 จาก 24 กรณีที่ศึกษา หรือ 75% จากกรณีที่ศึกษา และผลการศึกษาที่สำคัญของการวิจัยนี้คือ เมื่อข้อมูลทั้งสองกลุ่มมีความเบ้และมีความโด่งมากๆ ช่วงความเชื่อมั่นสำหรับอัตราส่วนค่ามัธยฐานของประชากรสองกลุ่มวิธีของ Price และ Bonett ที่ปรับร่วมกับวิธีบูตสแทร็ปที และวิธีบูตสแทร็ปเปอร์เซ็นต์ไทล์ มีประสิทธิภาพที่ดีกว่าวิธีของ Price และ Bonett ทุกกรณีที่ศึกษา

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Research Article

Confidence Intervals for a Ratio of Two Population Medians by Price and Bonett Bootstrap-t Method

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Abstract

The confidence intervals for a ratio of two population medians are proposed and compared with the approximate confidence interval of the Price and Bonett. The proposed methods are modified from the Bonett method with the bootstrap-t method and the bootstrap percentile method. The Monte Carlo simulation technique was performed 5,000 times repeatedly, along with a round of random sampling using the bootstrap method 5,000 times. The simulation involved both data from the population with the non-normal distribution and the small size of each sample by using the R program. The results indicated that for all studied distributions, the Price and Bonett bootstrap-t confidence interval and the bootstrap percentile confidence interval performance were superior to the Price and Bonett confidence interval in 18 out of 24 case studies or 75% of case studies. The most important result of this study was that both groups showed high skewness and high kurtosis, and the Price and Bonett bootstrap-t confidence interval and the bootstrap percentile confidence interval performance were superior to the Price and Bonett confidence interval in all the studied cases.

Keywords: Bootstrap, Median, Confidence Interval, Coverage Probability, Average Length

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1. Introduction

The median rather than the mean may be a more appropriate measure when the distribution of the response is skewed or non-normal, especially in a small size which usually has a non-normal distribution [1], [2]. In this study, the researcher considered design for the ratio of two population medians because the ratio of Medians may be very meaningful when the response variable is measured on a ratio scale such as measures of time, area, or volume [3], [4]. Price and Bonett [2] proposed 100(1–α)% distribution-free confidence interval for the ratio of two population medians and estimated the variance of the sample median by Price and Bonett's Method [5] which was developed from Hettmansperger and McKean's method [6] combined with Hettmansperger and Sheather's method [7] for simulation studies that confidence interval by Price and Bonett's method has a coverage probability close to confidence levels in some cases, especially when the sample size in both groups is 30. However, Price and Bonett's method did not perform well in many cases, for example, both groups had a positive skewed response and the sample size in both groups was 10.

In the opinion of the researcher, confidence interval for the ratio of two population medians by Price and Bonett's method didn't perform well in many cases due to using the standard normal distribution to estimate, which wasn't appropriate in the instance that the distribution of estimation for the ratio of two population medians were without normal distributions. Efron and Tibshirani [8] introduced the bootstrap method as a tool to estimate the standard error of a statistic and use this method to find the confidence interval for an unknown parameter. Tongkaw [1] carried out a study to develop confidence interval of different medians for two population by using integrated Price and Bonett's method combined with bootstrap method, and found in a simulation study that the proposed method had a coverage probability close to confidence levels. Many cases were included in Tongkaw's [9] study of developing a method to estimate the variance confidence interval by modifying bootstrap-t method with the confidence interval of variance for non-normal distribution by Bonett's method [10] and found in a simulation study that the proposed method performed well in many cases for positive skewed distribution.

Therefore, this study aimed to develop 100 (1–α)% distribution-free confidence intervals for the ratio of two population medians by modifying the bootstrap-t method with Price and Bonett's method which in this study was called "Price and Bonett with Bootstrap-t method". The coverage probability and average length of these confidence intervals consisted of 3 methods; Price and Bonett, Bootstrap percentile and Price and Bonett with Bootstrap-t, which were compared from Monte Carlo simulation by using the R program.

2. Materials and Methods

2.1 The Price and Bonett confidence interval

Let $Y_{(1)1} \le Y_{(2)1} \ldots \le Y_{(n_1)1}$ and $Y_{(1)2} \le Y_{(2)2} \ldots \le Y_{(n_2)2}$ be independent ordered and continuous random variables. Let $\hat{\theta}_1$ and $\hat{\theta}_2$ be the usual sample medians obtained from each sample which are estimations of the respective population medians, θ_1 and θ_2.

The Price and Bonett 100(1–α)% confidence intervals for the ratio of two population medians [2] is

$$\left(\frac{\hat{\theta}_1}{\hat{\theta}_2}\right)\exp\left\{\mp z_{\frac{\alpha}{2}}\left(var\left(\hat{\theta}_1\right)+var\left(\hat{\theta}_2\right)\right)^{\frac{1}{2}}\right\} \quad (1)$$

$$L_1=\left(\frac{\hat{\theta}_1}{\hat{\theta}_2}\right)\exp\left\{+z_{\frac{\alpha}{2}}\left(var\left(\hat{\theta}_1\right)+var\left(\hat{\theta}_2\right)\right)^{\frac{1}{2}}\right\}$$

$$U_1=\left(\frac{\hat{\theta}_1}{\hat{\theta}_2}\right)\exp\left\{-z_{\frac{\alpha}{2}}\left(var\left(\hat{\theta}_1\right)+var\left(\hat{\theta}_2\right)\right)^{\frac{1}{2}}\right\}$$

Where $var(\hat{\theta}_1)$ and $var(\hat{\theta}_2)$ is a distribution-free estimate of the variance of $\hat{\theta}_1$ and $\hat{\theta}_2$ for the confidence interval for θ_1/θ_2 [5] by the modified estimate of the variance of $\hat{\theta}$ between Hettmansperger and McKean's method [6] with Hettmansperger and Sheathe's method [7].

$$var\left(\hat{\theta}\right)=\left(\frac{(ln(Y_{(n_j-c_j+1)j})^{-ln(Y_{(c_j)j})})}{2z_j}\right);\ j=1,2 \quad (2)$$

Where $c_j=\frac{n_{j+1}}{2}-n_j^{1/2}$ is rounded to the nearest nonzero integer, $Z_j=\emptyset^{-1}(1-\frac{P_j}{2})$, and $P_j=\Sigma_{i=0}^{c_j-1}\{\frac{n!}{i!(n-1)!}\}(0.5)^{n-i}$. The Z_j is the $1-\frac{P_j}{2}$ quantile of the standard normal distribution. The $(L_1,\ U_1)$ is the lower confidence interval and upper confidence interval of this method.

2.2 Proposed confidence intervals

In this section, two new confidence intervals for the ratio of two population medians were proposed. The first proposed confidence interval was Price and Bonett with Bootstrap-t method.

Let $Y^*_{(1)1}\le Y^*_{(2)1}\ldots\le Y^*_{(n1)1}$ and $Y^*_{(1)2}\le Y^*_{(2)2}\ldots\le Y^*_{(n2)2}$ be independent ordered and continuous random variables when $n_1\le n_2$ by.

Let $\hat{\theta}_1$ and $\hat{\theta}_2$ be the usual sample medians obtained from each sample which are estimations of the respective population medians, θ_1 and θ_2.

The Price and Bonett with Bootstrap-t 100(1–α)% confidence intervals for the ratio of two population medians is

$$\left(\frac{\hat{\theta}_1}{\hat{\theta}_2}\right)\exp\left\{\mp t^*_{\frac{\alpha}{2}}\left(var\left(\hat{\theta}_1\right)+var\left(\hat{\theta}_2\right)\right)^{\frac{1}{2}}\right\} \quad (3)$$

$$t^*_k=\frac{\hat{\Delta}^*_k-\hat{\Delta}_B}{\sqrt{var\left(\hat{\Delta}^*\right)}};\ k=1,2,3\ldots,B \quad (4)$$

$$L_2=\left(\frac{\hat{\theta}_1}{\hat{\theta}_2}\right)\exp\left\{+t^*_{\frac{\alpha}{2}}\left(var\left(\hat{\theta}_1\right)+var\left(\hat{\theta}_2\right)\right)^{\frac{1}{2}}\right\}$$

$$U_2=\left(\frac{\hat{\theta}_1}{\hat{\theta}_2}\right)\exp\left\{-t^*_{\frac{\alpha}{2}}\left(var\left(\hat{\theta}_1\right)+var\left(\hat{\theta}_2\right)\right)^{\frac{1}{2}}\right\}$$

Where $var(\hat{\Delta}^*)$ is a distribution-free estimate of the variance of $\hat{\theta}k_1/\hat{\theta}k_2$ for the confidence interval for θ_1/θ_2 [5]. B is the number of repetitions by bootstrap method, $\hat{\Delta}_B$ is point estimations of the ratio of two population medians from the bootstrap method and $t^*_{\frac{\alpha}{2}}$ from the bootstrap-t method is used. The $(L_2,$ $U_2)$ is the lower confidence interval and upper confidence interval of this method.

The second proposed confidence interval for the ratio of two population medians was created using the bootstrap percentile method. Hence, Let $Y_{(1)1}\le Y_{(2)1}\ldots\le Y_{(n_1)1}$ and $Y_{(1)2}\le Y_{(2)2}\ldots\le Y_{(n_2)2}$ be independent ordered and continuous random variables when $n_1\le n_2$ by the bootstrap method.

Let $\hat{\theta}_1$ and $\hat{\theta}_2$ be the usual sample medians obtained from each sample which are estimations of the respective population medians, θ_1 and θ_2.

The bootstrap confidence interval for a 100(1–α)% confidence intervals for the ratio of two population medians is

$$\left(\hat{\Delta}^*_{\frac{\alpha}{2}}, \hat{\Delta}^*_{1-\frac{\alpha}{2}}\right) \quad (5)$$

Where $\hat{\Delta}^*_{\frac{\alpha}{2}}$ (the lower confidence interval: L_3) and $\hat{\Delta}^*_{1-\frac{\alpha}{2}}$ (the upper confidence interval: U_3) are the $(\alpha/2)100$th and $(1-\alpha/2)100$th percentile of the bootstrap sample ratio of medians [8].

3. Results

This section provides simulation studies for the coverage probabilities and the average lengths of confidence intervals as proposed in section 3 and the Price and Bonett confidence interval. The confidence level was 95%, 5,000 samples of size (n_1, n_2) = (5, 5), (10,10), (10, 30), (30, 10) and (30, 30) were generated from Log-normal distribution, Chi-square distribution, Exponential distribution, Weibull distribution and Normal distribution. For the bootstrap method, 5,000 samples were drawn from the original sample.

The different confidence intervals, their estimated coverage probabilities and average length were considered. For each of the methods considered, a (1–α)100% confidence interval denoted by (L, U) was obtained (based on M = 5,000 replicates). The estimated coverage probability and the average length are given by [1], [11].

$$Coverage\ Probability\ (CP) = \frac{\Sigma_i^M Coverage_i}{M} \quad (6)$$

Where $Coverage_i$ has a value of 1 when the confidence intervals cover the Ratio of Two Population Medians and $Coverage_i$ has a value of 0 when the confidence intervals do not cover the Ratio of Two Population Medians.

$$Average\ Length\ (AL) = \frac{\Sigma_{i=1}^M (U_{ji} - L_{ji})}{M}; j = 1,2,3 \quad (7)$$

The (L_{ji}, U_{ji}) is the lower confidence interval and upper confidence interval. The performance considerations of the confidence interval (6) are determined by coverage probabilities at closer to the confidence level (1–α) and then compared with the average length (7). If any methods to compare the coverage probabilities value have a close to the confidence level (1-alpha; 0.95), then consider the width of the average length by considering the shortest value of the average length.

The simulation results are reported in Tables 1–4. The simulation results from Table 1 and Figure 1 showed that when Y_{i1} and Y_{i2} were Log-normal distribution (high skewness and high kurtosis) for sample sizes (5, 5) (20, 30) and (10, 10) the Bootstrap Percentile confidence interval (5) has coverage probabilities closer to the confidence level compared to the other two confidence intervals including average lengths shorter than the Price and Bonett confidence interval (1). The Price and Bonett with Bootstrap-t confidence interval (3) had coverage probabilities closer to the confidence level compared to the other two confidence intervals including average length shorter than the Price and Bonett confidence interval (1) with sample sizes (10, 20) (10, 30) and (30, 30).

Table 1 The estimated coverage probability (CP) and the average length (AL) of a 95% confidence intervals for Y_{i1} ~LogN(0,1) and Y_{i2} ~LogN(0,1)

Distribution1	Distribution2	n_1	n_2	CP&AL	Price and Bonett (1)	Price and Bonett with Bootstrap-t (3)	Bootstrap Percentile (5)
LogN(0,1) [6.2,120]	LogN(0,1) [6.2,120]	5	5	CP	0.9656	0.8600	**0.9648**
				AL	9.1313	7.0240	**7.0242**
		10	10	CP	0.9732	0.9244	**0.9684**
				AL	3.7327	3.0744	**3.0736**
		10	20	CP	0.9636	**0.9410**	0.9254
				AL	2.8123	**1.9609**	1.9603
		10	30	CP	0.9630	**0.9448**	0.8738
				AL	2.6190	**1.5730**	1.5724
		20	30	CP	0.9568	0.9400	**0.9450**
				AL	1.7723	1.5095	**1.5095**
		30	30	CP	0.9576	**0.9432**	0.9632
				AL	1.5140	**1.4758**	0.9632

Note: The approximate skewness and kurtosis values of each distribution are listed in brackets, and the figures in bold show the best performance in each sample size.

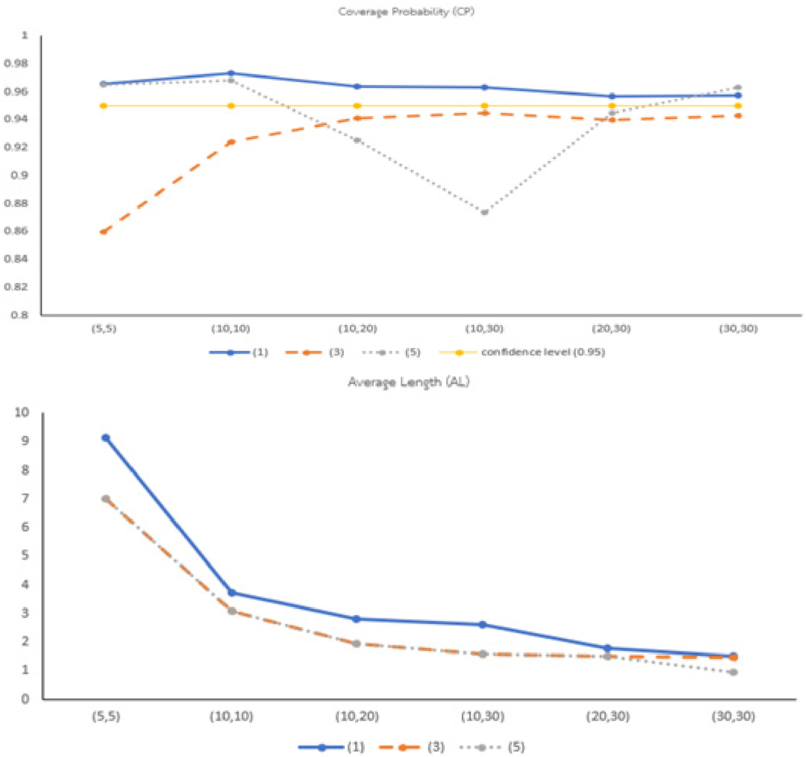

Figure 1 The estimated coverage probability (CP) and the average length (AL) of a 95% confidence intervals for Y_{i1} ~LogN(0,1) and Y_{i2} ~LogN(0,1).

The simulation results from Table 2 and Figure 2 showed that when Y_{i1} and Y_{i2} were Chi-square distribution (little skewness and little kurtosis) and Exponential distribution (medium skewness and medium kurtosis) respectively for sample sizes (5, 5), (10, 10) and (20,30) the Bootstrap Percentile confidence interval (5) had coverage probabilities closer to the confidence level compared with the other two confidence intervals including average lengths shorter than the Price and Bonett confidence interval (1). The Price and Bonett with Bootstrap-t confidence interval (3) had coverage probabilities closer to the confidence level compared with the other two confidence intervals including average shorter lengths than the Price and Bonett confidence interval (1) for sample size (10, 30). The Price and Bonett confidence interval (1) had coverage probabilities closer to the confidence level compared to the other two confidence intervals for sample sizes (10, 20) and (30, 30).

The simulation results from Table 3 and Figure 3 showed that when Y_{i1} and Y_{i2} were Log-normal distribution (high skewness and high kurtosis) and Chi-square distribution (low skewness and low kurtosis) respectively for sample sizes (5, 5) and (10, 10) the Bootstrap Percentile confidence interval (5) had coverage probabilities closer to the confidence level compared with the other two confidence intervals including average shorter lengths than the Price and Bonett confidence interval (1). The Price and Bonett with Bootstrap-t confidence interval (3) had coverage probabilities closer to the confidence level compared to the other two confidence intervals include average shorter lengths than the Price and Bonett confidence interval (1) for sample sizes (10, 20), (10, 30) and (20, 30). The Price and Bonett confidence interval (1) has coverage probabilities closer to the confidence level compared with the other two confidence intervals for sample size (30, 30).

Table 2 The estimated coverage probability (CP) and the average length (AL) of a 95% confidence intervals for Y_{i1} ~ ChiSq(4) and Y_{i2} ~ Exp(1)

Distribution1	Distribution2	n_1	n_2	CP&AL	Price and Bonett (1)	Price and Bonett with Bootstrap-t (3)	Bootstrap Percentile (5)
ChiSq(4) [1.4,6]	Exp(1) [2,9]	5	5	CP	0.9700	0.8386	**0.9658**
				AL	66.352	152.3082	**152.3081**
		10	10	CP	0.9744	0.8862	**0.9648**
				AL	19.5902	16.8246	**16.8205**
		10	20	CP	**0.9600**	0.9274	0.9350
				AL	**13.2468**	9.76354	9.7594
		10	30	CP	0.9630	**0.9424**	0.9080
				AL	10.5547	**7.5313**	7.5283
		20	30	CP	0.9600	0.9366	**0.9520**
				AL	7.8600	7.1971	**7.1984**
		30	30	CP	**0.9476**	0.9334	0.9618
				AL	**7.1872**	7.1521	7.1496

Note: The approximate skewness and kurtosis values of each distribution are listed in brackets, and the figures in bold show the best performance in each sample size.

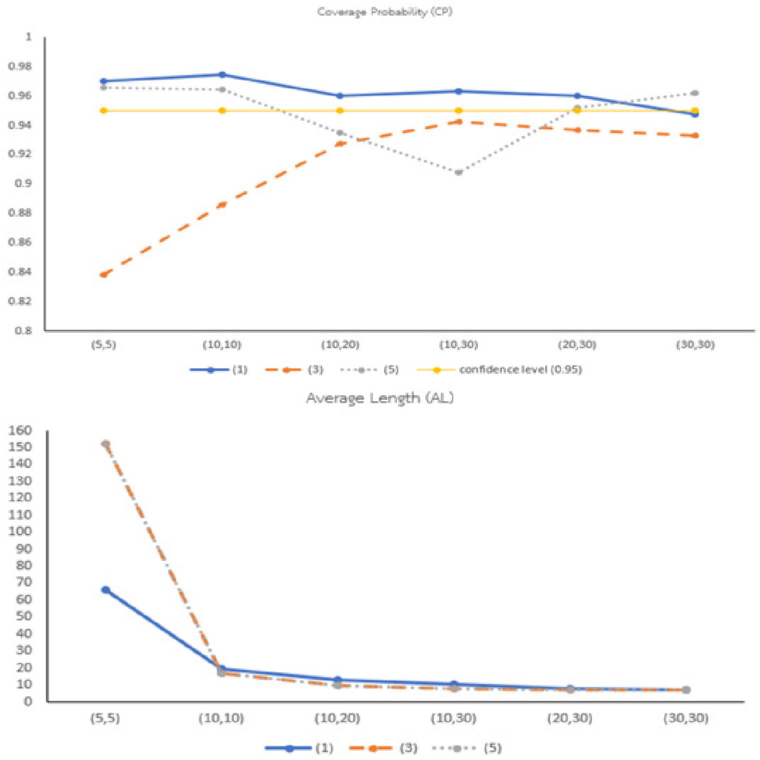

Figure 2 The estimated coverage probability (CP) and the average length (AL) of a 95% confidence intervals for Y_{i1} ~ ChiSq(4) and Y_{i2} ~ Exp(1).

Table 3 The estimated coverage probability (CP) and the average length (AL) of a 95% confidence intervals for Y_{i1} ~ LogN(0,1) and Y_{i2} ~ ChiSq(4)

Distribution1	Distribution2	n_1	n_2	CP&AL	Price and Bonett (1)	Price and Bonett with Bootstrap-t (3)	Bootstrap Percentile (5)
LogN(0,1) [6.2,120]	ChiSq(4) [1.4,6]	5	5	CP	0.9660	0.8668	**0.9668**
				AL	2.1072	2.0428	**2.0400**
		10	10	CP	0.9686	0.9156	**0.9596**
				AL	0.9310	0.8052	**0.8050**
		10	20	CP	0.9604	**0.9374**	0.9024
				AL	0.7874	**0.5311**	0.5308
		10	30	CP	0.9652	**0.9456**	0.8486
				AL	0.7351	**0.4171**	0.4169
		20	30	CP	0.9542	**0.9450**	0.9376
				AL	0.4665	**0.3937**	0.3935
		30	30	CP	**0.9538**	0.9400	0.9634
				AL	**0.3878**	0.3824	0.3828

Note: The approximate skewness and kurtosis values of each distribution are listed in brackets, and the figures in bold show the best performance in each sample size.

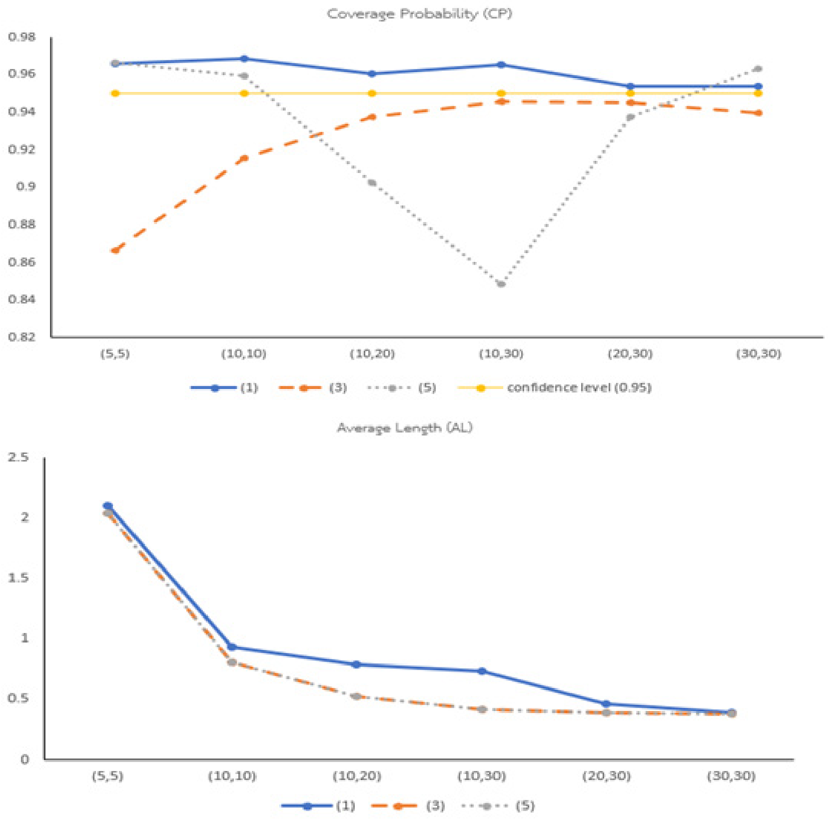

Figure 3 The estimated coverage probability (CP) and the average length (AL) of a 95% confidence intervals for Y_{i1} ~ LogN(0,1) and Y_{i2} ~ ChiSq(4).

The simulation results from Table 4 and Figure 4 showed that when Y_{i1} and Y_{i2} were Log-normal distribution (high skewness and high kurtosis) and Exponential distribution (medium skewness and medium kurtosis) respectively for samples size (5, 5) the Price and Bonett confidence interval (1) had coverage probabilities closer to the confidence level and average shorter lengths compared to the other two confidence intervals. The Bootstrap Percentile confidence interval (5) had coverage probabilities closer to the confidence level compared to the other two confidence intervals including average shorter length than the Price and Bonett confidence interval (1) for sample sizes (10, 10) and (10, 20). The Price and Bonett with Bootstrap-t confidence interval (3) have coverage probabilities closer to the confidence level compared to the other two confidence intervals including average shorter length than the Price and Bonett confidence interval (1) for sample size (10, 30). The Price and Bonett confidence interval (1) had coverage probabilities closer to the confidence level compared to the other two confidence intervals for sample size (30, 30). Finally, when the sample size of both groups increased, coverage probabilities increased closer to the confidence level, and average length reduced.

Table 4 The estimated coverage probability (CP) and the average length (AL) of a 95% confidence intervals for Y_{i1} ~ LogN(0,1) and Y_{i2} ~ Exp(1)

Distribution1	Distribution2	n_1	n_2	CP&AL	Price and Bonett (1)	Price and Bonett with Bootstrap-t (3)	Bootstrap Percentile (5)
LogN(0,1) [6.2,120]	Exp(1) [2,9]	5	5	CP	**0.9646**	0.7856	0.9646
				AL	**25.7500**	63.4833	63.4833
		10	10	CP	0.9726	0.8604	**0.9624**
				AL	7.0460	6.4607	**6.4580**
		10	20	CP	0.9660	0.9238	**0.9326**
				AL	4.6790	3.5046	**3.5052**
		10	30	CP	0.9634	**0.9376**	0.8898
				AL	4.0748	**2.6469**	2.6459
		20	30	CP	0.9556	0.9316	**0.9524**
				AL	2.8015	2.4974	**2.4965**
		30	30	CP	**0.9566**	0.9282	0.9640
				AL	**2.4665**	2.4658	2.4651

Note: The approximate skewness and kurtosis values of each distribution are listed in brackets, and the figures in bold show the best performance in each sample size.

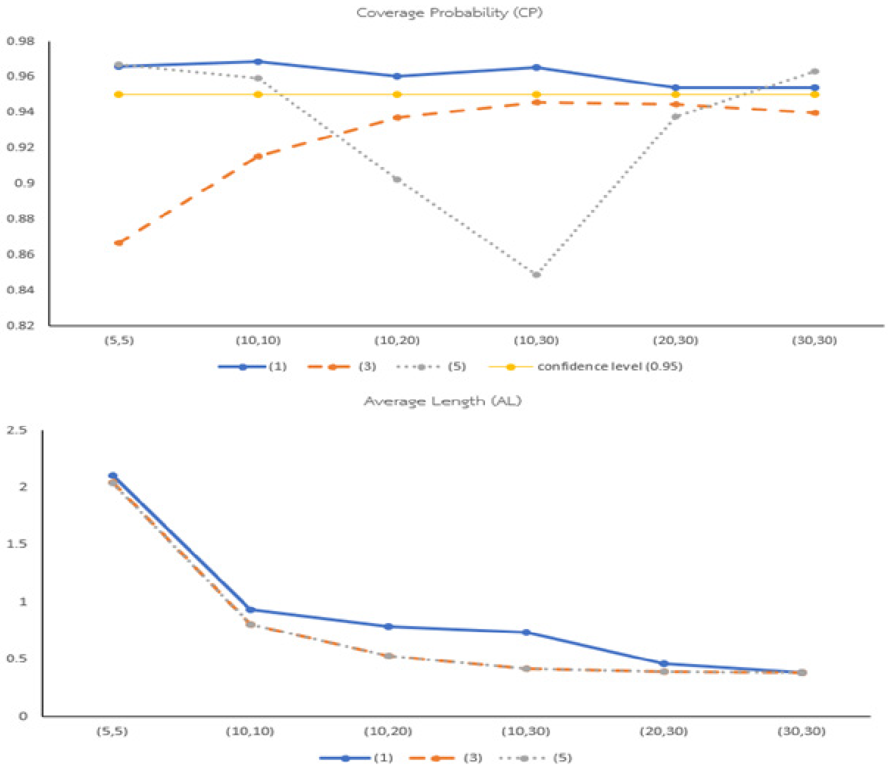

Figure 4 The estimated coverage probability (CP) and the average length (AL) of a 95% confidence intervals for Y_{i1} ~ LogN(0,1) and Y_{i2} ~ Exp(1).

4. Conclusions and Discussion

This study aimed to develop 100(1−α)% distribution-free confidence intervals for the ratio of two population medians by modifying the bootstrap-t method with Price and Bonett's method [2] which in this study is called "Price and Bonett with Bootstrap-t method". The Price and Bonett with Bootstrap-t confidence interval (3) (The developed method in this study), the Bootstrap Percentile confidence interval (5) (proposed in this study) and the Price and Bonett confidence interval (1) (original method in this study) for a ratio of two population medians were studied and compared. Considering coverage probabilities and average lengths, the Price and Bonett with Bootstrap-t confidence interval was the superior performance method compared with the other two methods, in 8 case studies especially when the sample size is (10, 30). The Bootstrap Percentile confidence interval was the best method compared with the other two methods, in 11 case studies especially when the sample size is (10, 10). The Price and Bonett confidence interval was the best method compared with the other two methods, in 5 case studies especially when the sample size is (30, 30) with no high non-normal distribution. The results indicated that for all the studied distributions, the Price and Bonett bootstrap-t confidence interval and the bootstrap percentile confidence interval performance were better than the Price and Bonett confidence interval in 18 out of 24 case studies or 75% of case studies. The most important result of this study was that both groups showed high skewness and high kurtosis, and the Price and Boonett bootstrap-t confidence interval and the bootstrap percentile confidence interval had a better performance than the Price and Bonett confidence interval in all the cases studied.

The results of this study correspond to the results of Tongkaw [1] conservation results, which found that the confidence interval of difference medians for two populations for free-distribution by Price Bonett Bootstrap-t method showed results better than the Price Bonett method, in many case studies corresponding to the results of Tongkaw [9] conservation results, which found that the confidence interval of variance by Bonett with bootstrap-t performed better than the original method in many case studies. Finally, as the sample size of both groups increased, coverage probabilities increased closer to the confidence level and average lengths reduced corresponding to conservation results of Panichkitkosolkul [11], Tongkaw [9] and Tongkaw [1]. Their study results show that the sample size of increased coverage probabilities increased closer to the confidence level and average lengths reduced. However, this is a study from Log-normal distribution, Chi-square distribution, Exponential distribution, Weibull distribution and Normal distribution, which still lacks additional studies for other distributions especially non-normal distributions.

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